CUET (UG) – MATHEMATICS

Chapter Test - Unit III: Calculus - Continuity and Differentiability

SOLUTIONS

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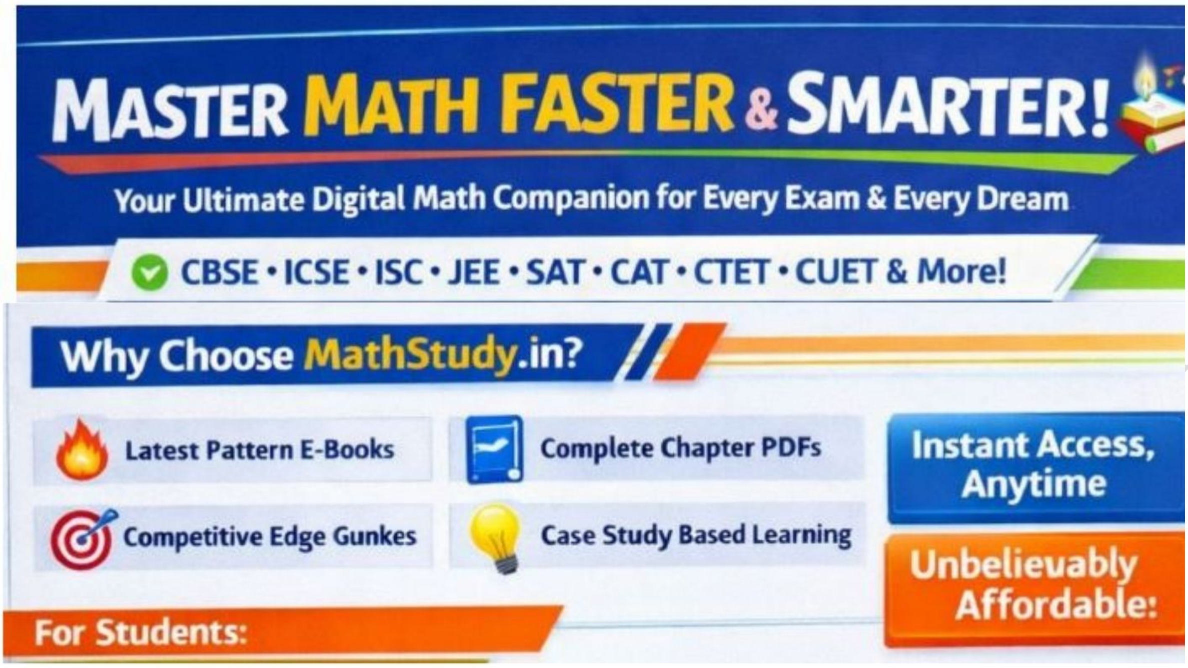

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Solutions

1. **Correct Option: (A)**. Let $u = \cos^{-1}(2x^2 - 1)$ and $v = \cos^{-1} x$. Substitute $x = \cos\theta$. Then $u = \cos^{-1}(\cos 2\theta) = 2\theta$ and $v = \theta$. So $u = 2v \implies \frac{du}{dv} = 2$.

2. **Correct Option: (B)**. $y = \frac{1}{2}[\log(1 - \sin x) - \log(1 + \sin x)]$. $y' = \frac{1}{2}[\frac{-\cos x}{1-\sin x} - \frac{\cos x}{1+\sin x}] = \frac{-\cos x}{2}[\frac{1+\sin x+1-\sin x}{1-\sin^2 x}] = \frac{-\cos x}{2} \cdot \frac{2}{\cos^2 x} = -\sec x$.

3. **Correct Option: (B)**. $\lim_{x\to\pi/2} \frac{k\cos x}{\pi-2x}$ (L'Hopital's) $= \lim_{x\to\pi/2} \frac{-k\sin x}{-2} = k/2$. Given limit is 3, so $k/2 = 3 \implies k = 6$.

4. **Correct Option: (B)**. Taking log: $m\log x + n\log y = (m + n)\log(x + y)$. Differentiating: $\frac{m}{x} + \frac{n}{y}y' = \frac{m+n}{x+y}(1 + y')$. Solving for y' gives y/x.

5. **Correct Option: (B)**. Put $x = \tan\theta$. $\tan^{-1}(\frac{\sec\theta-1}{\tan\theta}) = \tan^{-1}(\frac{1-\cos\theta}{\sin\theta}) = \tan^{-1}(\tan\theta/2) = \theta/2$. So $u = \frac{1}{2}\tan^{-1} x$. Derivative w.r.t $\tan^{-1} x$ is $1/2$.

6. **Correct Option: (A)**. $y' = a^x \log a + ax^{a-1}$. At $x = a$, $y'(a) = a^a \log a + a \cdot a^{a-1} = a^a \log a + a^a = a^a(1 + \log a)$.

7. **Correct Option: (B)**. $\frac{dy}{dx} = \frac{a\sin\theta}{a(1+\cos\theta)} = \tan(\theta/2)$. $\frac{d^2y}{dx^2} = \frac{d}{d\theta}[\tan(\theta/2)] \cdot \frac{d\theta}{dx} = \frac{1}{2}\sec^2(\theta/2) \cdot \frac{1}{a(1+\cos\theta)}$. At $\theta = \pi/2$, $\frac{1}{2}(2) \cdot \frac{1}{a(1+0)} = 1/a$ (Re-check: $\frac{1}{2}\sec^2(\pi/4) \cdot \frac{1}{a} = \frac{1}{2} \cdot 2 \cdot \frac{1}{a} = 1/a$). Correction: Option (A).

8. **Correct Option: (B)**. $y = 3\tan^{-1} x$. So $y' = \frac{3}{1+x^2}$.

9. **Correct Option: (A)**. $y' = \frac{1}{x+\sqrt{x^2+a^2}} \cdot (1 + \frac{x}{\sqrt{x^2+a^2}}) = \frac{1}{x+\sqrt{x^2+a^2}} \cdot \frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}} = \frac{1}{\sqrt{x^2+a^2}}$.

10. **Correct Option: (B)**. $e^x+e^y = e^x e^y \implies \frac{1}{e^y}+\frac{1}{e^x} = 1 \implies e^{-y}+e^{-x} = 1$. Differentiating: $-e^{-y}y' - e^{-x} = 0 \implies y' = -\frac{e^{-x}}{e^{-y}} = -e^{y-x}$.

11. **Correct Option: (A)**. $\frac{d(\sin^2 x)}{dx} = \sin 2x$. $\frac{d((\log x)^2)}{dx} = \frac{2\log x}{x}$. Ratio $= \frac{x\sin 2x}{2\log x}$.

12. **Correct Option: (B)**. Standard result for $y = \sin(m\sin^{-1} x)$ is $(1-x^2)y_2-xy_1+m^2y = 0$. So result is $-m^2y$.

13. **Correct Option: (A)**. $y = \cos^{-1}(\frac{1-(x^n)^2}{1+(x^n)^2}) = 2\tan^{-1}(x^n)$. $y' = \frac{2}{1+(x^n)^2} \cdot nx^{n-1} = \frac{2nx^{n-1}}{1+x^{2n}}$.

14. **Correct Option: (B)**. $f'(x) = \frac{1}{\log x} \cdot \frac{1}{x}$. $f'(e) = \frac{1}{1 \cdot e} = 1/e$.

15. **Correct Option: (B)**. $x^2(1 + y) = y^2(1 + x) \implies (x - y)(x + y + xy) = 0$. Since $x \neq y$, $y = \frac{-x}{1+x}$. Applying quotient rule gives $y' = -1/(1 + x)^2$.

16. **Correct Option: (B)**. $\frac{d}{dx}(x^x) = x^x(1 + \log x)$. At $x = e$, $e^e(1 + \log e) = e^e(1 + 1) = 2e^e$.

17. **Correct Option: (B)**. $y = \sin^{-1} x + \cos^{-1} x = \pi/2$. Derivative of constant is 0.

18. **Correct Option: (D)**. The series is the expansion of e^x. So $y = e^x$ and $\frac{dy}{dx} = e^x = y$.

19. **Correct Option: (B)**. $y = \tan^{-1} a - \tan^{-1} x$. $y' = 0 - \frac{1}{1+x^2}$.

20. **Correct Option: (A)**. In the 2nd quadrant $(3\pi/4)$, $\cos x$ is negative, so $y = -\cos x$. $y' = \sin x$. $\sin(3\pi/4) = 1/\sqrt{2}$.