CUET Mathematics Test

Chapter: Continuity and Differentiability

SOLUTIONS

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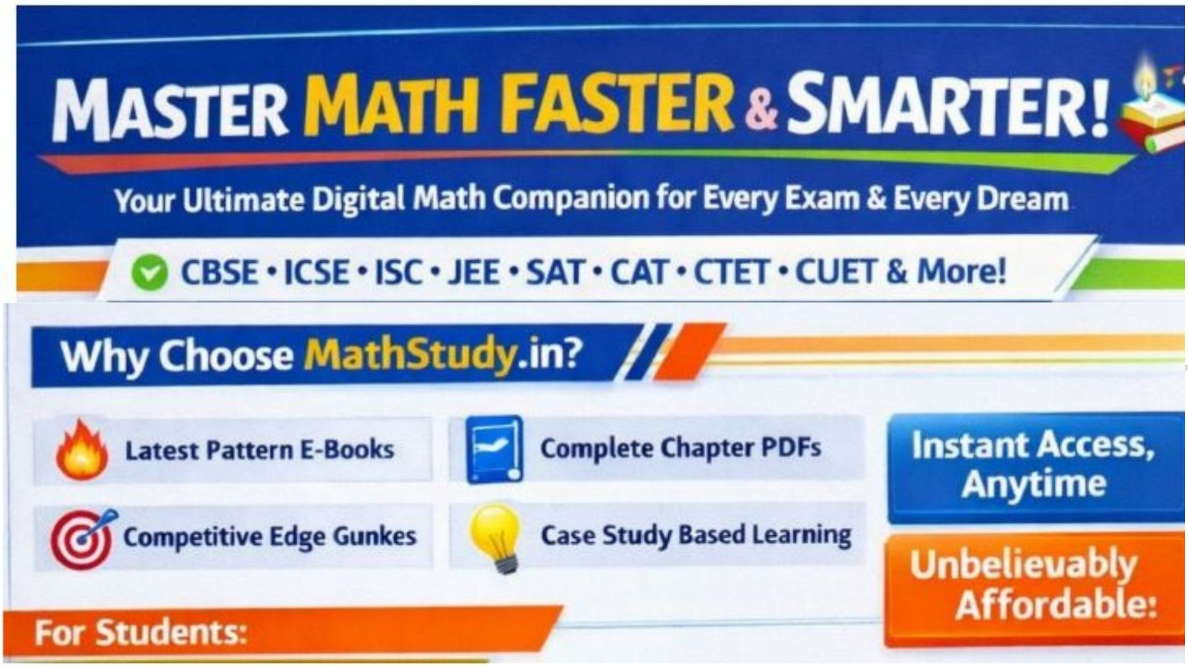

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Solutions

1. **Solution:** Modulus functions $|x - a|$ are not differentiable at $x = a$. Therefore, $|x - 1|$ is not differentiable at $x = 1$ and $|x - 2|$ is not differentiable at $x = 2$. **Correct Option: (C)**

2. **Solution:** $y = \tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right) = \tan^{-1}(\tan(\pi/4 + x)) = \pi/4 + x$. Differentiating gives $dy/dx = 1$. **Correct Option: (C)**

3. **Solution:** $dx/d\theta = a(1 + \cos\theta)$ and $dy/d\theta = a\sin\theta$. $dy/dx = \frac{\sin\theta}{1+\cos\theta}$. At $\theta = \pi/2$, $dy/dx = \frac{1}{1+0} = 1$. **Correct Option: (A)**

4. **Solution:** $y = \sqrt{\sin x + y} \Rightarrow y^2 = \sin x + y$. Differentiating, $2yy' = \cos x + y' \Rightarrow y'(2y-1) = \cos x \Rightarrow y' = \frac{\cos x}{2y-1}$. **Correct Option: (A)**

5. **Solution:** Let $u = \sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1} x$. Let $v = \cos^{-1}(1 - 2x^2) = 2\sin^{-1} x$ (since $x > 0$). $du/dv = 1$. **Correct Option: (A)**

6. **Solution:** $y = e^{x+y} \Rightarrow \log y = x + y$. Differentiating: $\frac{1}{y}y' = 1 + y' \Rightarrow y'(\frac{1}{y} - 1) = 1 \Rightarrow y' = \frac{y}{1-y}$. **Correct Option: (A)**

7. **Solution:** $f(x) = \frac{\log(\log x)}{\log x}$. Using quotient rule and evaluating at $x = e$ where $\log e = 1$ and $\log(\log e) = 0$, we get $f'(e) = 1/e$. **Correct Option: (B)**

8. **Solution:** $y\log x = x - y \Rightarrow y(1 + \log x) = x \Rightarrow y = \frac{x}{1+\log x}$. Use quotient rule: $y' = \frac{(1+\log x)-1}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2}$. **Correct Option: (A)**

9. **Solution:** $y' = \frac{1}{\sqrt{1-x^2}}$. $y'' = \frac{x}{(1-x^2)^{3/2}}$. Substituting into the expression: $(1-x^2)\frac{x}{(1-x^2)^{3/2}} - x\frac{1}{\sqrt{1-x^2}} = \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}} = 0$. **Correct Option: (C)**

10. **Solution:** Greatest integer function is discontinuous at all integers. At $x = 1.5$, it is a constant 1, so it is both continuous and differentiable. **Correct Option: (D)**

11. **Solution:** $y = \log\sqrt{\frac{2\sin^2(x/2)}{2\cos^2(x/2)}} = \log\tan(x/2)$. $y' = \frac{1}{\tan(x/2)}\sec^2(x/2)\cdot\frac{1}{2} = \frac{\cos(x/2)}{\sin(x/2)}\frac{1}{\cos^2(x/2)}\frac{1}{2} = \frac{1}{2\sin(x/2)\cos(x/2)} = \frac{1}{\sin x} = \csc x$. **Correct Option: (C)**

12. **Solution:** $dy/dx = \frac{2a}{2at} = 1/t$. $\frac{d^2y}{dx^2} = \frac{d}{dt}(1/t)\cdot\frac{dt}{dx} = \frac{-1}{t^2}\cdot\frac{1}{2at} = \frac{-1}{2at^3}$. **Correct Option: (C)**

13. **Solution:** $\frac{d(e^{\sin x})}{d(\cos x)} = \frac{e^{\sin x}\cdot\cos x}{-\sin x} = -e^{\sin x}\cot x$. **Correct Option: (A)**

14. **Solution:** Put $x = \tan\theta$. $y = \tan^{-1}(\frac{\sec\theta - 1}{\tan\theta}) = \tan^{-1}(\frac{1-\cos\theta}{\sin\theta}) = \tan^{-1}(\tan(\theta/2)) = \theta/2 = \frac{1}{2}\tan^{-1} x$. $y' = \frac{1}{2(1+x^2)}$. **Correct Option: (A)**

15. **Solution:** $f'(x) = e^x g(x) + e^x g'(x)$. $f'(0) = e^0 g(0) + e^0 g'(0) = 1(2) + 1(1) = 3$. **Correct Option: (B)**

16. **Solution:** Taking log: $\log y = x\log\sin x$. Diff: $\frac{1}{y}y' = 1\cdot\log\sin x + x\cdot\frac{\cos x}{\sin x}$. $y' = y[\log\sin x + x\cot x]$. **Correct Option: (A)**

17. **Solution:** Standard result for implicit functions of the form $x^m y^n = (x + y)^{m+n}$. Taking log and differentiating always leads to y/x. **Correct Option: (B)**

18. **Solution:** LHD at $x = 0$ is -1. RHD at $x = 0$ is 1. Since LHD $\neq$ RHD, it is not differentiable. **Correct Option: (D)**

19. **Solution:** Put $x = \tan\theta$. $y = \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1} x$. $y' = \frac{2}{1+x^2}$. **Correct Option: (A)**

20. **Solution:** $f'(x) = \cos x - \sin x$. $f''(x) = -\sin x - \cos x$. At $x = \pi/4$: $f''(\pi/4) = -1/\sqrt{2} - 1/\sqrt{2} = -2/\sqrt{2} = -\sqrt{2}$. **Correct Option: (C)**