CUET Mathematics Test

Chapter: Unit II: Algebra (Matrices and Determinants)

SOLUTIONS

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Solutions

1. **Solution:** $(I+A)^3 - 7A = I^3 + 3I^2A + 3IA^2 + A^3 - 7A$. Given $A^2 = A$, so $A^3 = A^2 \cdot A = A \cdot A = A$. Expression becomes $I + 3A + 3A + A - 7A = I + 7A - 7A = I$. **Correct Option: (C)**

2. **Solution:** $A_{2\times3} \times B_{3\times2} = (AB)_{2\times2}$. The transpose of a 2×2 matrix is also 2×2. **Correct Option: (B)**

3. **Solution:** For a singular matrix, $|A| = 0$. $2(5) - 3(k) = 0 \implies 10 - 3k = 0 \implies k = 10/3$. **Correct Option: (A)**

4. **Solution:** For a matrix of order n, $|adj(A)| = |A|^{n-1}$. Here $n = 3$, so $|adj(A)| = 5^{3-1} = 5^2 = 25$. **Correct Option: (B)**

5. **Solution:** Unique solution exists if $|A| \neq 0$. $|A| = 1(k+2) - 1(2k+3) + 1(4-3) = k + 2 - 2k - 3 + 1 = -k$. For unique solution, $-k \neq 0 \implies k \neq 0$. **Correct Option: (A)**

6. **Solution:** $A^TA = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \sin^2\theta + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$. **Correct Option: (B)**

7. **Solution:** Let $C = AB - BA$. $C^T = (AB - BA)^T = (AB)^T - (BA)^T = B^TA^T - A^TB^T$. Since A, B are symmetric, $A^T = A, B^T = B$. $C^T = BA - AB = -(AB - BA) = -C$. Thus, it is skew-symmetric. **Correct Option: (B)**

8. **Solution:** For order n, $|kA| = k^n|A|$. Here $n = 3, k = 2$, so $|2A| = 2^3|A| = 8 \times 2 = 16$. **Correct Option: (C)**

9. **Solution:** $|A| = 4 - 6 = -2$. $adj(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$. $A^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$. **Correct Option: (A)**

10. **Solution:** Area $= \frac{1}{2}|2(1-8) - 7(1-10) + 1(8-10)| = \frac{1}{2}|2(-7) - 7(-9) + 1(-2)| = \frac{1}{2}|-14 + 63 - 2| = \frac{1}{2}|47| = 23.5$. **Correct Option: (B)**

11. **Solution:** $AA^{-1} = I \implies det(A \cdot A^{-1}) = det(I) \implies det(A) \cdot det(A^{-1}) = 1 \implies det(A^{-1}) = 1/det(A)$. **Correct Option: (B)**

12. **Solution:** $x + y = 6, xy = 8$. These are roots of $t^2 - 6t + 8 = 0$, which are 4 and 2. $5 + z = 5 \implies z = 0$. So $(4, 2, 0)$ or $(2, 4, 0)$. **Correct Option: (C)**

13. **Solution:** $A^2 = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$. Thus $A^3 = A^4 = \cdots = O$. $f(A) = I + A + O + \cdots = I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. **Correct Option: (A)**

14. **Solution:** Determinant of a skew-symmetric matrix of odd order is always 0. **Correct Option: (C)**

15. **Solution:** This is the condition for an inconsistent system with no solution. **Correct Option: (C)**

16. **Solution:** $a_{ji} = j - i = -(i - j) = -a_{ij}$. Therefore $A^T = -A$, making it skew-symmetric. **Correct Option: (B)**

17. **Solution:** For 2×2 matrix, interchange diagonal elements and change signs of off-diagonal elements. **Correct Option: (A)**

18. **Solution:** $(A - B)(A + B) = A^2 + AB - BA - B^2$. For this to equal $A^2 - B^2$, $AB - BA$ must be O, so $AB = BA$. **Correct Option: (A)**

19. **Solution:** Trace is sum of diagonal elements: $1 + 5 + 9 = 15$. **Correct Option: (A)**

20. **Solution:** Use Mathematical Induction or verify for $k = 1, 2$. For $k = 1$: $\begin{bmatrix} 1+2 & -4 \\ 1 & 1-2 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$. **Correct Option: (A)**

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