CUET Mathematics Test

Chapter: Unit III: Calculus (Applied Mathematics)

SOLUTIONS

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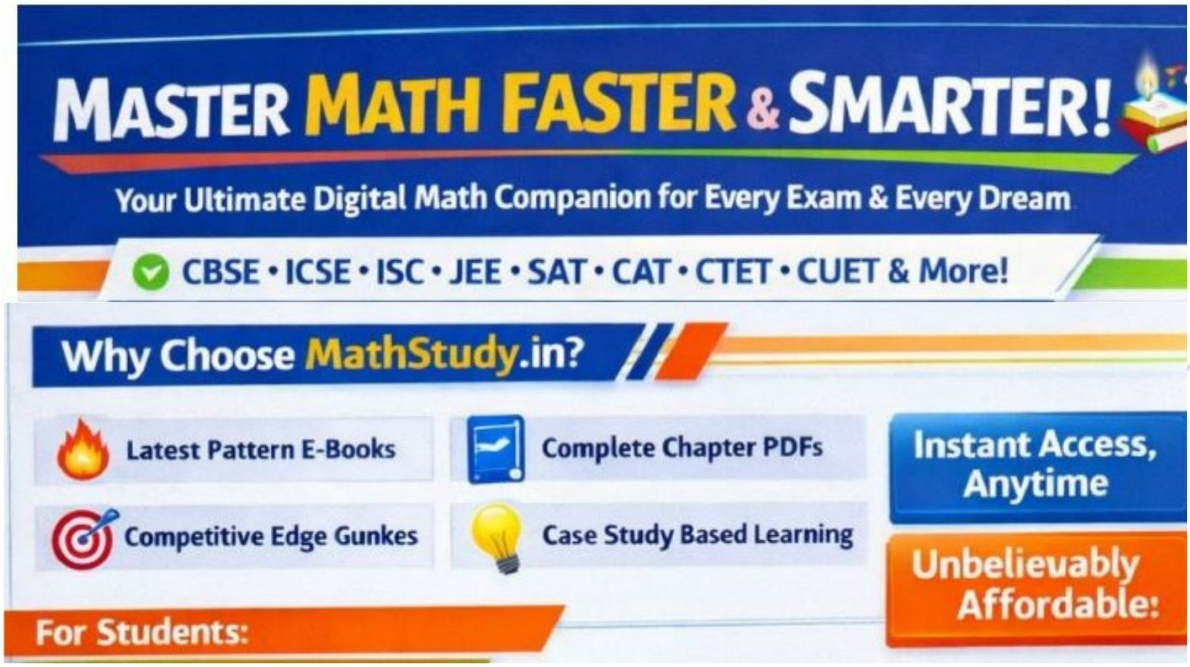

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Solutions

1. **Solution:** $y' = ae^{ax}\sin bx + be^{ax}\cos bx$. $y'' = a^2e^{ax}\sin bx + abe^{ax}\cos bx + abe^{ax}\cos bx - b^2e^{ax}\sin bx$. Substituting into the equation $y'' - 2ay' + (a^2+b^2)y = 0$. **Correct Option: (A)**

2. **Solution:** $MC = C'(x) = x^2 - 10x + 30$. To minimize MC, find $\frac{d(MC)}{dx} = 2x - 10 = 0 \implies x = 5$. **Correct Option: (C)**

3. **Solution:** $MR = R'(x) = 20 - x$. At $x = 10$, $MR = 20 - 10 = 10$. **Correct Option: (A)**

4. **Solution:** $f'(x) = 3x^2 - 12x + 12 = 3(x-2)^2$. Since $(x-2)^2 \geq 0$ for all x, $f'(x) \geq 0$. The function is strictly increasing. **Correct Option: (A)**

5. **Solution:** $f'(x) = (1-x)^2 + x \cdot 2(1-x)(-1) = (1-x)(1-x-2x) = (1-x)(1-3x)$. Critical points $x = 1, 1/3$. $f(1/3) = (1/3)(2/3)^2 = 4/27$. **Correct Option: (A)**

6. **Solution:** $30 = 50 - 2x \implies 2x = 20 \implies x_0 = 10$. $CS = \int_0^{10}(50 - 2x)dx - (30 \times 10) = [50x - x^2]_0^{10} - 300 = (500 - 100) - 300 = 100$. **Correct Option: (B)**

7. **Solution:** At $x_0 = 4$, $p_0 = 10 + 3(4) = 22$. $PS = (22 \times 4) - \int_0^4 (10 + 3x)dx = 88 - [10x + 1.5x^2]_0^4 = 88 - (40 + 24) = 88 - 64 = 24$. **Correct Option: (A)**

8. **Solution:** Using partial fractions: $\frac{1}{x(1+x^2)} = \frac{1}{x} - \frac{x}{1+x^2}$. $\int_1^2(\frac{1}{x} - \frac{x}{1+x^2})dx = [\log x - \frac{1}{2}\log(1+x^2)]_1^2 = (\log 2 - \frac{1}{2}\log 5) - (0 - \frac{1}{2}\log 2) = \frac{3}{2}\log 2 - \frac{1}{2}\log 5 = \frac{1}{2}\log(8/5)$. **Correct Option: (B)**

9. **Solution:** This is a Bernoulli equation or homogeneous. Let $y = vx$. $v + x\frac{dv}{dx} = v + v^2 \implies \frac{dv}{v^2} = \frac{dx}{x} \implies -1/v = \log x + c \implies -x/y = \log x + c$. Rearranging gives $y = \frac{x}{c - \log x}$. **Correct Option: (A)**

10. **Solution:** $P = P_0e^{kt}$. $2P_0 = P_0e^{k(50)} \implies 2 = e^{50k} \implies 50k = \log 2 \implies k = \frac{\log 2}{50}$. **Correct Option: (A)**

11. **Solution:** $R(x) = \int(20 - 4x - 3x^2)dx = 20x - 2x^2 - x^3 + C$. Since $R(0) = 0$, $C = 0$. **Correct Option: (A)**

12. **Solution:** $f'(x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6) = 6(x-2)(x-3)$. $f'(x) < 0$ when $2 < x < 3$. **Correct Option: (A)**

13. **Solution:** $f'(x) = 1 - 1/x^2$. $f'(x) = 0 \implies x = \pm 1$. $f''(x) = 2/x^3$. $f''(1) = 2 > 0$ (min). Min value $f(1) = 1 + 1 = 2$. **Correct Option: (A)**

14. **Solution:** $\int_0^a \frac{1}{1+(2x)^2}dx = [\frac{1}{2}\tan^{-1}(2x)]_0^a = \frac{1}{2}\tan^{-1}(2a) = \frac{\pi}{8} \implies \tan^{-1}(2a) = \frac{\pi}{4} \implies 2a = 1 \implies a = 1/2$. **Correct Option: (A)**

15. **Solution:** Due to the term $\sin(dy/dx)$, the differential equation is not a polynomial in its derivatives. **Correct Option: (D)**

16. **Solution:** $VC = \int(4 + 0.1x)dx = 4x + 0.05x^2$. $TC = 4x + 0.05x^2 + 50$. For $x = 10$: $TC = 40 + 5 + 50 = 95$. **Correct Option: (A)**

17. **Solution:** Points of intersection: $x^2 = x \implies x = 0, 1$. Area $= \int_0^1(x - x^2)dx = [x^2/2 - x^3/3]_0^1 = 1/2 - 1/3 = 1/6$. **Correct Option: (A)**

18. **Solution:** $A = A_0e^{-kt}$. $1/2 = e^{-kT} \implies -kT = \log(1/2) = -\log 2 \implies k = \frac{\log 2}{T}$. $A = A_0e^{-(t\log 2)/T}$. **Correct Option: (A)**

19. **Solution:** By Leibniz rule, $f'(x) = \sqrt{2-x^2}$. At $x = 1$, $f'(1) = \sqrt{2-1} = 1$. **Correct Option: (A)**

20. **Solution:** $f'(x) = 3x^2 - 6x + 5$, $f''(x) = 6x - 6$. $f''(x) = 0 \implies x = 1$. $f(1) = 1 - 3 + 5 = 3$. Point is $(1, 3)$. **Correct Option: (A)**

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