SOLUTIONS

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Solutions

1. **Solution:** $y = x^3 \log x$. First derivative: $\frac{dy}{dx} = x^3(\frac{1}{x}) + 3x^2 \log x = x^2 + 3x^2 \log x$. Second derivative: $\frac{d^2y}{dx^2} = 2x + 3[x^2(\frac{1}{x}) + 2x \log x] = 2x + 3x + 6x \log x = 5x + 6x \log x$.

2. **Solution:** $f'(x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6) = 6(x-2)(x-3)$. For strictly increasing, $f'(x) > 0$. By the sign scheme, this occurs when $x \in (-\infty, 2) \cup (3, \infty)$.

3. **Solution:** $f'(x) = \cos x - \sin x$. Setting $f'(x) = 0 \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$. Evaluating at boundaries and critical point: $f(0) = 1$, $f(\frac{\pi}{4}) = \sqrt{2}$, $f(\pi) = -1$. Max value is $\sqrt{2}$ and min value is -1.

4. **Solution:** $\frac{dy}{dx} = mAe^{mx} + nBe^{nx}$ and $\frac{d^2y}{dx^2} = m^2Ae^{mx} + n^2Be^{nx}$. Substituting these into the differential equation: $(m^2Ae^{mx} + n^2Be^{nx}) - (m+n)(mAe^{mx} + nBe^{nx}) + mn(Ae^{mx} + Be^{nx}) = 0$.

5. **Solution:** $f'(x) = \frac{1}{2} - \frac{2}{x^2}$. Setting $f'(x) = 0 \Rightarrow x^2 = 4 \Rightarrow x = 2$ (since $x > 0$). $f''(x) = \frac{4}{x^3}$. $f''(2) = \frac{4}{8} > 0$, indicating a Local Min at $x = 2$. Local min value $f(2) = 2$.

6. **Solution:** $f'(x) = 4 - x$. Critical point $x = 4$. Comparing values: $f(-2) = -8 - 2 = -10$; $f(4) = 16 - 8 = 8$; $f(4.5) = 18 - 10.125 = 7.875$. Absolute max is 8, Absolute min is -10.

7. **Solution:** $f'(x) = (x-2)^2 + x \cdot 2(x-2) = (x-2)(x-2+2x) = (x-2)(3x-2)$. For $f(x)$ to be increasing, $f'(x) \geq 0$. This occurs in $x \in (-\infty, \frac{2}{3}] \cup [2, \infty)$.

8. **Solution:** $f'(x) = \frac{1}{1+(\sin x + \cos x)^2} \cdot (\cos x - \sin x)$. In the interval $(0, \frac{\pi}{4})$, $\cos x > \sin x$, therefore $f'(x) > 0$, making it strictly increasing.

9. **Solution:** $f'(x) = 2\cos 2x - 1 = 0 \Rightarrow \cos 2x = \frac{1}{2} \Rightarrow x = \pm\frac{\pi}{6}$. Using $f''(x) = -4\sin 2x$: $f''(\frac{\pi}{6}) < 0$ (Max), $f''(-\frac{\pi}{6}) > 0$ (Min).

10. **Solution:** $\frac{dy}{dx} = \frac{a \sin\theta}{a(1+\cos\theta)} = \tan\frac{\theta}{2}$. Differentiating again with respect to x: $\frac{d^2y}{dx^2} = \frac{1}{2}\sec^2\frac{\theta}{2} \cdot \frac{1}{a(1+\cos\theta)}$. At $\theta = \frac{\pi}{2}$, value is $1/a$.

11. **Solution:** Let θ be the vertical angle. Area $A = 2R^2 \sin\theta(1 + \cos\theta)$. Differentiating A with respect to θ and setting to zero gives $\theta = \pi/3$, implying the triangle is equilateral.

12. **Solution:** Minimize $D^2 = x^2 + (y-c)^2 = y + (y-c)^2$. Let $g(y) = y^2 + y(1-2c) + c^2$. Setting $g'(y) = 0$ gives $y = c - 1/2$. Shortest distance is $\sqrt{c - 1/4}$ for $c \geq 1/2$.

13. **Solution:** $f'(x) = \frac{1}{1+x} - \frac{4}{(2+x)^2} = \frac{x^2}{(1+x)(2+x)^2}$. Since $x^2 \geq 0$ and the denominator is positive for $x > -1$, $f'(x) \geq 0$, thus $f(x)$ is increasing.

14. **Solution:** $y' = e^x(\sin 5x + 5\cos 5x)$. Differentiating again: $y'' = e^x(\sin 5x + 5\cos 5x) + e^x(5\cos 5x - 25\sin 5x) = e^x(10\cos 5x - 24\sin 5x)$.

15. **Solution:** Let parts be x and $15 - x$. $P(x) = x^2(15-x)^3$. $P'(x) = x(15-x)^2[30 - 5x]$. Setting $P'(x) = 0$ gives $x = 6$. The parts are 6 and 9.

16. **Solution:** $f'(x) = \cos x + \sin x$. For strictly decreasing, $f'(x) < 0 \Rightarrow \tan x < -1$. In $(0, 2\pi)$, this interval is $(\frac{3\pi}{4}, \frac{7\pi}{4})$.

17. **Solution:** $\sqrt{1-x^2}\frac{dy}{dx} = 2\sin^{-1} x$. Squaring gives $(1-x^2)(\frac{dy}{dx})^2 = 4y$. Differentiating again and dividing by $2\frac{dy}{dx}$ yields $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$.

18. **Solution:** Volume $V = \pi(R^2h - h^3/4)$. Setting $dV/dh = 0$ gives $h = 2R/\sqrt{3}$. Substituting h back into V gives the maximum volume as $\frac{4\pi R^3}{3\sqrt{3}}$.

19. **Solution:** $y' = 3x^2 - 1$. Local minimum occurs at $x = 1/\sqrt{3}$ where $y'' = 6x > 0$. The slope of the tangent y' at this point is $3(1/3) - 1 = 0$.

20. **Solution:** $xy = 96 \Rightarrow P = 2(x + 96/x)$. $P'(x) = 2(1 - 96/x^2) = 0 \Rightarrow x = 4\sqrt{6}$. Since $x = y$, the rectangle is a square with side $4\sqrt{6}$.