CUET Mathematics Test

Chapter: Applications of Derivatives

SOLUTIONS

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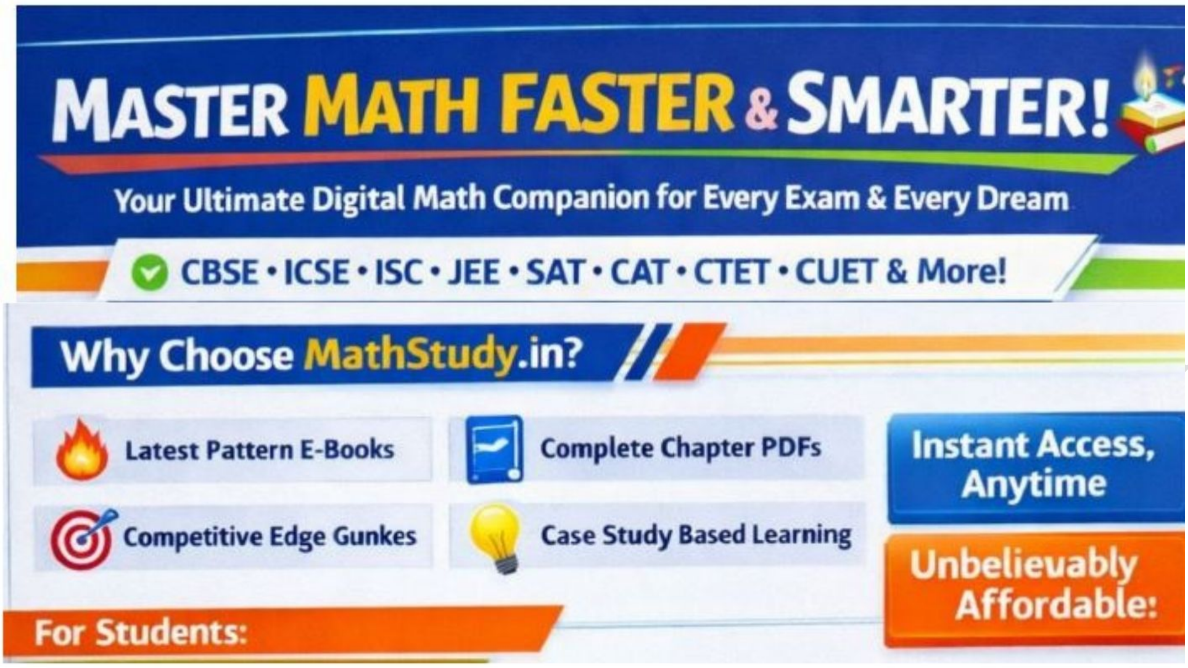

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Solutions

1. **Solution:** Volume $V = \frac{4}{3}\pi r^3$. Given $dV/dt = 900$. $dV/dt = 4\pi r^2(dr/dt)$. $900 = 4\pi(15)^2(dr/dt) \Rightarrow 900 = 900\pi(dr/dt) \Rightarrow dr/dt = 1/\pi$. **Correct Option: (A)**

2. **Solution:** Marginal Revenue $MR = dR/dx = 6x + 36$. When $x = 5$, $MR = 6(5) + 36 = 66$. **Correct Option: (A)**

3. **Solution:** $f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x-3)(x+2)$. For strictly decreasing, $f'(x) < 0 \Rightarrow (x-3)(x+2) < 0 \Rightarrow x \in (-2, 3)$. **Correct Option: (A)**

4. **Solution:** $y' = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2-x)$. For increasing, $y' > 0$. Since $e^{-x} > 0$, we need $x(2-x) > 0 \Rightarrow x \in (0, 2)$. **Correct Option: (D)**

5. **Solution:** By AM-GM inequality, $(ax + b/x)/2 \geq \sqrt{ax \cdot b/x} = \sqrt{ab} \Rightarrow ax + b/x \geq 2\sqrt{ab}$. **Correct Option: (B)**

6. **Solution:** $dy/dx = 3x^2 - 1$. At $x = 2$, $dy/dx = 3(4) - 1 = 11$. **Correct Option: (B)**

7. **Solution:** $dy/dx = 2x$. For tangent parallel to x-axis, $dy/dx = 0 \Rightarrow 2x = 0 \Rightarrow x = 0$. Then $y = 0^2 = 0$. **Correct Option: (B)**

8. **Solution:** Let $f(x) = \sin x + \cos x$. $f'(x) = \cos x - \sin x = 0 \Rightarrow \tan x = 1 \Rightarrow x = \pi/4$. $f(\pi/4) = 1/\sqrt{2} + 1/\sqrt{2} = \sqrt{2}$. **Correct Option: (C)**

9. **Solution:** $dy/dx = e^x$. Slope of line $y = x$ is 1. $e^x = 1 \Rightarrow x = 0$. Then $y = e^0 = 1$. **Correct Option: (A)**

10. **Solution:** $A = \pi r^2$. $dA/dt = 2\pi r(dr/dt)$. Given $dr/dt = 4$ and $r = 10$, $dA/dt = 2\pi(10)(4) = 80\pi$. **Correct Option: (C)**

11. **Solution:** $f'(x) = \sec^2 x - 1 = \tan^2 x$. Since $\tan^2 x \geq 0$, $f(x)$ is always increasing. **Correct Option: (A)**

12. **Solution:** Let $y = (1/x)^x \Rightarrow \ln y = -x \ln x$. $y'/y = -(\ln x + 1)$. $y' = 0 \Rightarrow \ln x = -1 \Rightarrow x = 1/e$. Max value is $(e)^{1/e}$. **Correct Option: (B)**

13. **Solution:** Minimize $D^2 = x^2 + (y-5)^2 = 2y + (y-5)^2$. $f(y) = y^2 - 8y + 25$. $f'(y) = 2y - 8 = 0 \Rightarrow y = 4$. Then $x^2 = 8 \Rightarrow x = 2\sqrt{2}$. **Correct Option: (A)**

14. **Solution:** $f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$. $f''(x) = 6x - 12$. $f''(1) = -6 < 0$ (Max), $f''(3) = 6 > 0$ (Min). **Correct Option: (A)**

15. **Solution:** $A = \pi r^2 \Rightarrow dA/dr = 2\pi r$. At $r = 6$, $dA/dr = 12\pi$. **Correct Option: (B)**

16. **Solution:** $y^2 = 4x \Rightarrow 2yy' = 4 \Rightarrow y' = 2/y$. Given slope of tangent is 1. $2/y = 1 \Rightarrow y = 2$. Then $2^2 = 4x \Rightarrow x = 1$. **Correct Option: (A)**

17. **Solution:** $f'(x) = 3x^2 - 36x + 96 = 3(x-4)(x-8)$. Critical points $x = 4, 8$. $f(0) = 0, f(4) = 160, f(8) = 128, f(9) = 135$. Minimum is 0. **Correct Option: (B)**

18. **Solution:** $v = ds/dt = 3t^2 - 12t + 12$. $a = dv/dt = 6t - 12$. $a = 0 \Rightarrow t = 2$. $v = 3(4) - 12(2) + 12 = 0$. **Correct Option: (B)**

19. **Solution:** $f'(x) = k - \cos x$. For increasing, $f'(x) \geq 0 \Rightarrow k \geq \cos x$. Since max value of $\cos x$ is 1, $k \geq 1$. **Correct Option: (C)**

20. **Solution:** $dy/dt = 2(dx/dt)$. Since $y = \sqrt{x}$, $dy/dt = \frac{1}{2\sqrt{x}}(dx/dt)$. So $\frac{1}{2\sqrt{x}} = 2 \Rightarrow \sqrt{x} = 1/4 \Rightarrow x = 1/16$. Then $y = 1/4$. **Correct Option: (A)**