CUET Mathematics Test

Chapter: Integration and its Applications

SOLUTIONS

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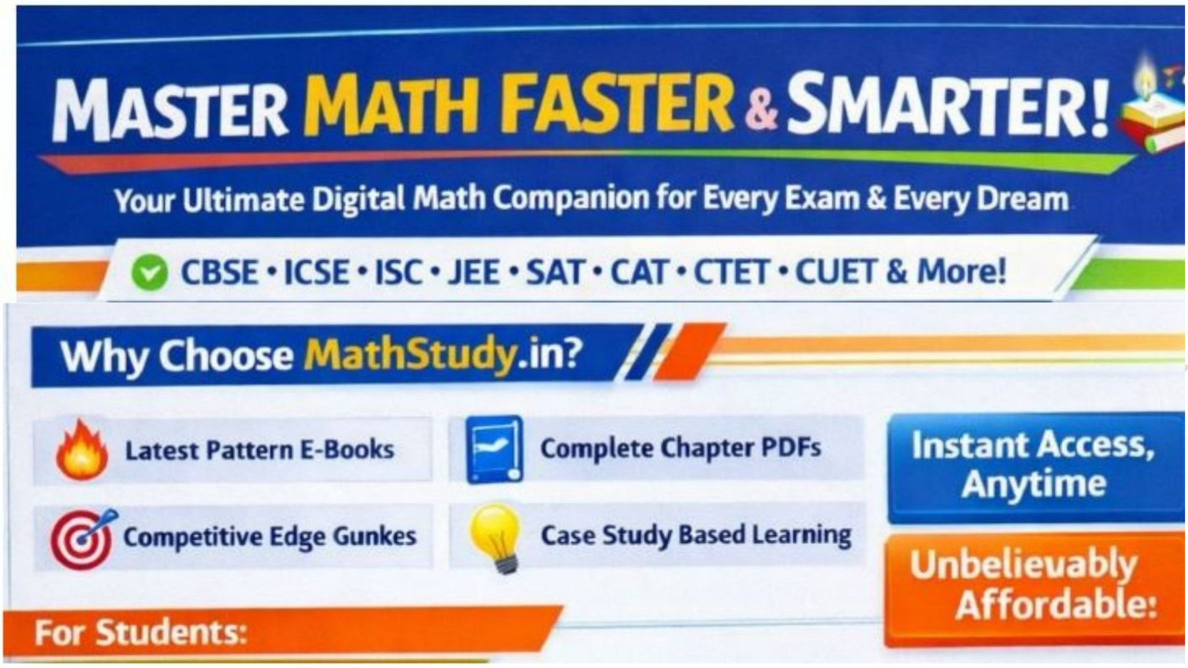

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Solutions

1. **Solution:** Using $\cos 2x = 2\cos^2 x - 1$, the integral becomes $\int \frac{2(\cos^2 x - \cos^2 \alpha)}{\cos x - \cos \alpha} dx = \int 2(\cos x + \cos \alpha) dx$. Correct Answer: $2(\sin x + x \cos \alpha) + C$.

2. **Solution:** Write $1 + \sin x = (\cos \frac{x}{2} + \sin \frac{x}{2})^2$. The integral becomes $\int \frac{dx}{\cos \frac{x}{2} + \sin \frac{x}{2}}$. Divide numerator and denominator by $\cos \frac{x}{2}$ to get $\sqrt{2} \int \frac{dx/2}{\cos(x/2 - \pi/4)}$. Correct Answer: $\sqrt{2} \log |\tan(\frac{x}{4} + \frac{\pi}{8})| + C$.

3. **Solution:** Divide by x^2: $\int \frac{1+1/x^2}{x^2+1/x^2} dx$. Let $x - 1/x = t$, then $(1 + 1/x^2)dx = dt$ and $x^2 + 1/x^2 = t^2 + 2$. Integral is $\int \frac{dt}{t^2+(\sqrt{2})^2}$. Correct Answer: $\frac{1}{\sqrt{2}} \tan^{-1}(\frac{x^2-1}{\sqrt{2}x}) + C$.

4. **Solution:** Multiply numerator and denominator by x^{n-1}. Let $x^n + 1 = t$, then $nx^{n-1}dx = dt$. Integral is $\frac{1}{n} \int \frac{dt}{(t-1)t} = \frac{1}{n} \int (\frac{1}{t-1} - \frac{1}{t}) dt$. Correct Answer: $\frac{1}{n} \log |\frac{x^n}{x^n+1}| + C$.

5. **Solution:** Let $x - a = t \Rightarrow x = t + a$. Integral becomes $\int \frac{\sin(t+a)}{\sin t} dt = \int (\frac{\sin t \cos a + \cos t \sin a}{\sin t}) dt = \int (\cos a + \sin a \cot t) dt$. Correct Answer: $(x - a)\cos a + \sin a \log |\sin(x - a)| + C$.

6. **Solution:** Use property $\int_0^a f(x)dx = \int_0^a f(a - x)dx$. Let I be the integral. Adding I to itself gives $2I = \int_0^{\pi/2} 1 dx = \pi/2$. Correct Answer: $\pi/4$.

7. **Solution:** Use $x \to \pi - x$. $2I = \pi \int_0^\pi \frac{\sin x}{1+\cos^2 x} dx$. Let $\cos x = t$. $2I = \pi \int_{-1}^1 \frac{dt}{1+t^2} = \pi[\tan^{-1} t]_{-1}^1 = \pi(\pi/4 - (-\pi/4)) = \pi^2/2$. Correct Answer: $\pi^2/4$.

8. **Solution:** The function $x \cos(\pi x)$ is negative on $(-1, -1/2)$ and $(1/2, 1)$, and positive on $(-1/2, 1/2)$. Split the integral and evaluate. Correct Answer: $2/\pi + 1/\pi^2$.

9. **Solution:** Let $x = \tan \theta, dx = \sec^2 \theta d\theta$. Limits 0 to $\pi/4$. $I = \int_0^{\pi/4} \log(1 + \tan \theta) d\theta$. Using property $f(a - x)$, $2I = \int_0^{\pi/4} \log 2 d\theta$. Correct Answer: $\frac{\pi}{8} \log 2$.

10. **Solution:** Using property $\int_0^a f(x)dx = \int_0^a f(a - x)dx$. $2I = \int_0^\pi \log(1 - \cos^2 x)dx = \int_0^\pi \log(\sin^2 x)dx = 2\int_0^\pi \log(\sin x)dx$. Correct Answer: $-\pi \log 2$.

11. **Solution:** $y = 3\sqrt{x}$. Area $= \int_0^4 3\sqrt{x} dx = 3[\frac{x^{3/2}}{3/2}]_0^4 = 2[4^{3/2}] = 16$. Correct Answer: 16 sq. units.

12. **Solution:** $y = \frac{3}{4}\sqrt{16 - x^2}$. Total Area $= 4 \times \int_0^4 \frac{3}{4}\sqrt{16 - x^2} dx = 3[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4}]_0^4$. Correct Answer: 12π sq. units.

13. **Solution:** Area $= \int_0^{\pi/2} \cos x dx + |\int_{\pi/2}^{3\pi/2} \cos x dx| + \int_{3\pi/2}^{2\pi} \cos x dx = 1 + 2 + 1 = 4$. Correct Answer: 4 sq. units.

14. **Solution:** Area $= 2\int_0^4 \sqrt{y} dy = 2[\frac{y^{3/2}}{3/2}]_0^4 = \frac{4}{3}(8) = 32/3$. Correct Answer: 32/3 sq. units.

15. **Solution:** Let $xe^x = t \Rightarrow (xe^x + e^x)dx = dt$. Integral is $\int \sec^2 t dt = \tan t + C$. Correct Answer: $\tan(xe^x) + C$.

16. **Solution:** Points of intersection $(0, 0)$ and $(4a, 4a)$. Area $= \int_0^{4a} (\sqrt{4ax} - \frac{x^2}{4a}) dx = [\frac{2\sqrt{4a}}{3} x^{3/2} - \frac{x^3}{12a}]_0^{4a} = \frac{32a^2}{3} - \frac{16a^2}{3}$. Correct Answer: $16a^2/3$ sq. units.

17. **Solution:** Use property $x \to \pi/4 - x$. $\log(1 + \tan(\pi/4 - x)) = \log(1 + \frac{1-\tan x}{1+\tan x}) = \log(\frac{2}{1+\tan x})$. $I = \int \log 2 - I$. Correct Answer: $\frac{\pi}{8} \log 2$.

18. **Solution:** Multiply by $\frac{\sin(a-b)}{\sin(a-b)}$. Use $\sin(a-b) = \sin((x-b)-(x-a))$. Correct Answer: $\frac{1}{\sin(a-b)} \log|\frac{\sin(x-a)}{\sin(x-b)}| + C$.

19. **Solution:** Let $I = \int_0^{\pi/2} \log(\sin x)dx$. Using property, $I = \int_0^{\pi/2} \log(\cos x)dx$. $2I = \int_0^{\pi/2} \log(\sin x \cos x)dx = \int_0^{\pi/2} \log(\frac{\sin 2x}{2})dx$. Correct Answer: $-\frac{\pi}{2} \log 2$.

20. **Solution:** $y = x^2$ for $x \geq 0$ and $-x^2$ for $x < 0$. Area $= \int_{-1}^{0} -x^2 dx + \int_0^1 x^2 dx = |[-x^3/3]_{-1}^{0}| + [x^3/3]_0^1 = 1/3 + 1/3$. Correct Answer: 2/3 sq. units.