SOLUTIONS

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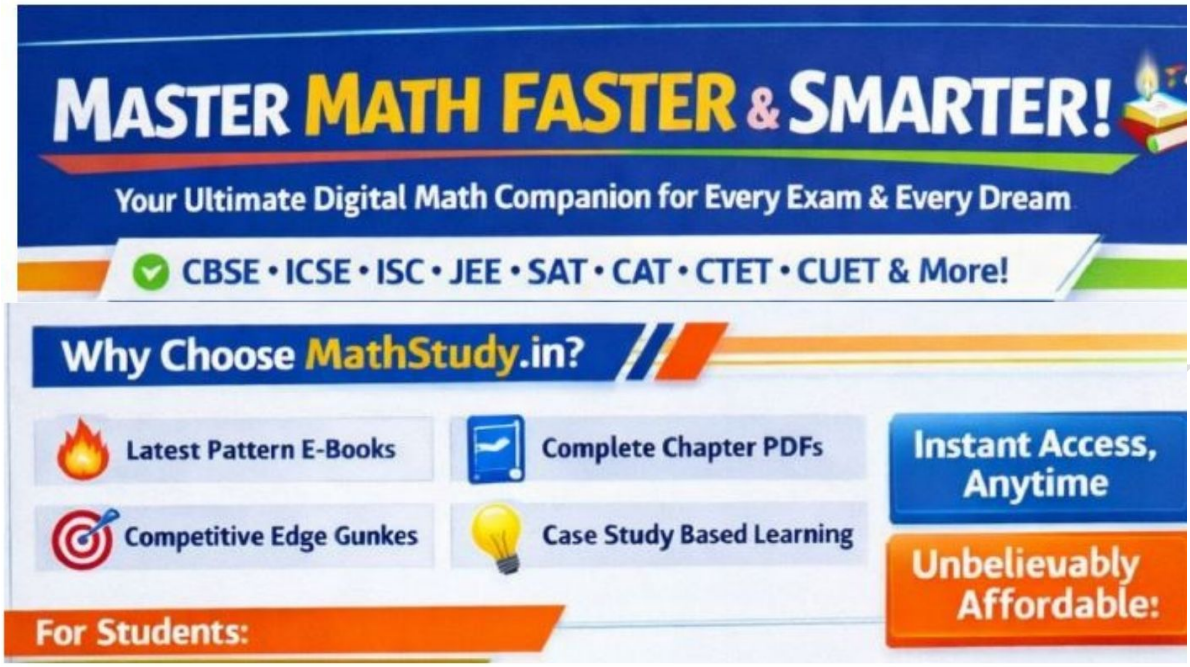

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Solutions

1. **Solution:** Equating corresponding elements: $2x + y = 5$ and $x - 2y = 0$. From the second equation, $x = 2y$. Substituting into the first: $2(2y) + y = 5 \Rightarrow 5y = 5 \Rightarrow y = 1$. Thus, $x = 2(1) = 2$. **Answer:** $x = 2, y = 1$.

2. **Solution:** $A^T = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$. Calculating $A^T A$:
$$\begin{bmatrix} \cos^2\alpha + \sin^2\alpha & -\cos\alpha\sin\alpha + \sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha + \cos\alpha\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I.$$

3. **Solution:** Let $P = A + A^T$. $P^T = (A + A^T)^T = A^T + (A^T)^T = A^T + A = P$. Since $P^T = P$, it is symmetric. Let $Q = A - A^T$. $Q^T = (A - A^T)^T = A^T - A = -(A - A^T) = -Q$. Since $Q^T = -Q$, it is skew-symmetric.

4. **Solution:** $3X = 5B - 2A = 5\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ -5 & 10 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -11 & 2 \end{bmatrix}$.
Thus, $X = \begin{bmatrix} -2/3 & 1/3 \\ -11/3 & 2/3 \end{bmatrix}$.

5. **Solution:** Perform $C_3 \to C_3 + C_2 \Rightarrow \Delta = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$. Factor out $(a+b+c)$ from C_3: $\Delta = (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$. Since C_1 and C_3 are identical, $\Delta = 0$.

6. **Solution:** Using the property $|kA| = k^n|A|$ for a matrix of order n. Here $n = 3$ and $k = 2$. $|2A| = 2^3 \times |A| = 8 \times 4 = 32$. **Answer: 32**.

7. **Solution:** $|A| = (2)(3) - (5)(1) = 6 - 5 = 1$. The matrix of cofactors is $\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$.
$adj(A) = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$. $A^{-1} = \frac{1}{|A|}adj(A) = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$.

8. **Solution:** $A = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}, B = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$. $|A| = -4 - 3 = -7$.
$A^{-1} = \frac{1}{-7}\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}$. $X = A^{-1}B = \frac{1}{-7}\begin{bmatrix} -10+3 \\ -5-2 \end{bmatrix} = \frac{1}{-7}\begin{bmatrix} -7 \\ -7 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.
Answer: $x = 1, y = 1$.

9. **Solution:** $AB = \begin{bmatrix} 2(0)+(-1)(-1) & 2(4)+(-1)(7) \\ 3(0)+2(-1) & 3(4)+2(7) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & 26 \end{bmatrix}$.
$(AB)^T = \begin{bmatrix} 1 & -2 \\ 1 & 26 \end{bmatrix}$.

10. **Solution:** $A^T = -A$. Taking determinants: $|A^T| = |-A|$. Since order is 3, $|-A| = (-1)^3|A| = -|A|$.
Since $|A^T| = |A|$, we have $|A| = -|A| \Rightarrow 2|A| = 0 \Rightarrow |A| = 0$.

11. **Solution:** Expanding along R_1: $|A| = 1(1-3) - 0(2-0) + (-1)(2-0) = 1(-2) - 0 - 2 = -4$.
Answer: -4.

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12. **Solution:** $A(adjA) = |A|I$. Taking determinants: $|A(adjA)| = ||A|I|$.
$|A||adjA| = |A|^n|I| \Rightarrow |A||adjA| = |A|^n$. Dividing by $|A|$ (as A is non-singular): $|adjA| = |A|^{n-1}$.

13. **Solution:** $Area = \frac{1}{2}\left|\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix}\right| = 3$. Expanding along R_2: $\frac{1}{2}|(-1)(0-3k)| = 3$.
$\frac{1}{2}|3k| = 3 \Rightarrow |k| = 2 \Rightarrow k = \pm 2$. **Answer:** $k = 2$ **or** -2.

14. **Solution:** $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$.
$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$.

15. **Solution:** $A^T = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$. Symmetric $P = \frac{1}{2}(A + A^T) = \begin{bmatrix} 1 & 5.5 \\ 5.5 & 7 \end{bmatrix}$.
Skew-symmetric $Q = \frac{1}{2}(A - A^T) = \begin{bmatrix} 0 & -0.5 \\ 0.5 & 0 \end{bmatrix}$. $A = P + Q$.

16. **Solution:** $|A| = 4 - 3 = 1$. $A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$.
$X = A^{-1}B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 8-9 \\ -4+6 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. **Answer:** $x = -1, y = 2$.

17. **Solution:** $|A| = ad - bc$. $adj(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.
$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

18. **Solution:** $(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = A(I)A^{-1} = AA^{-1} = I$.
Since the product of (AB) and $(B^{-1}A^{-1})$ is I, $(AB)^{-1} = B^{-1}A^{-1}$.

19. **Solution:** Writing in $AX = B$ form, the coefficient matrix $|A| = 9$.
Using Cramer's rule or $X = A^{-1}B$ yields $x = 1, y = 2, z = 3$. **Answer:** $x = 1, y = 2, z = 3$.

20. **Solution:** a_{21} is 4. Minor $M_{21} = \begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} = 18 - 24 = -6$.
Cofactor $C_{21} = (-1)^{2+1}M_{21} = -1(-6) = 6$. **Answer: Minor = -6, Cofactor = 6**.

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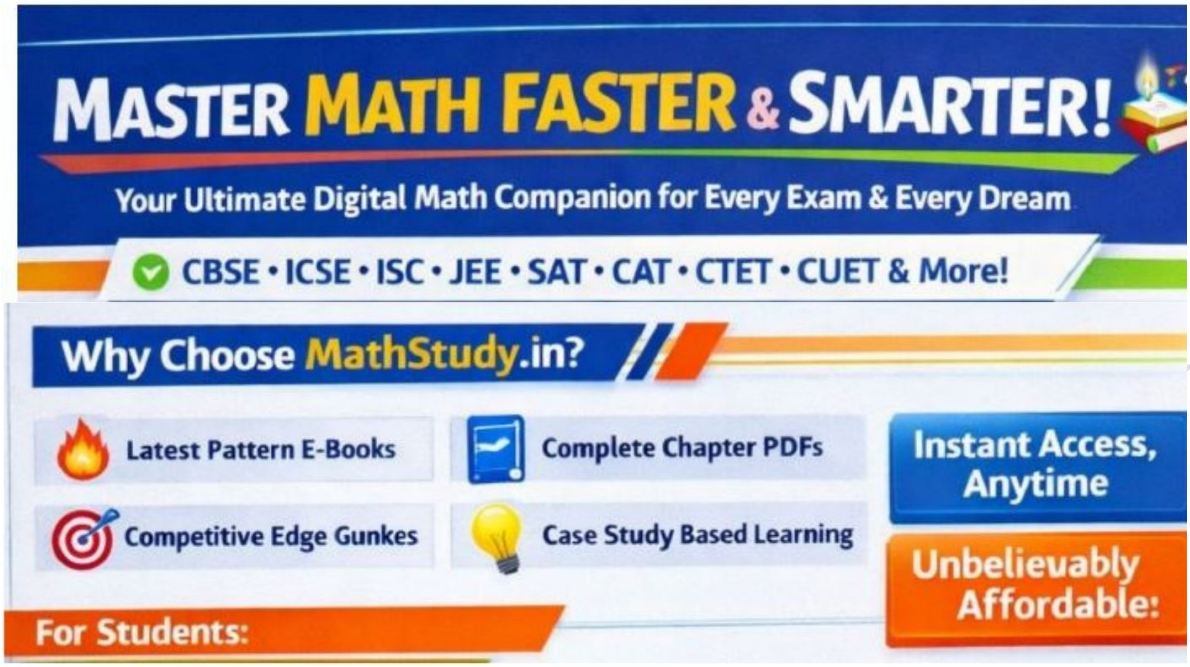

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