CUET (UG) – MATHEMATICS

Chapter Test - Unit III: Calculus - Applications of Derivatives

SOLUTIONS

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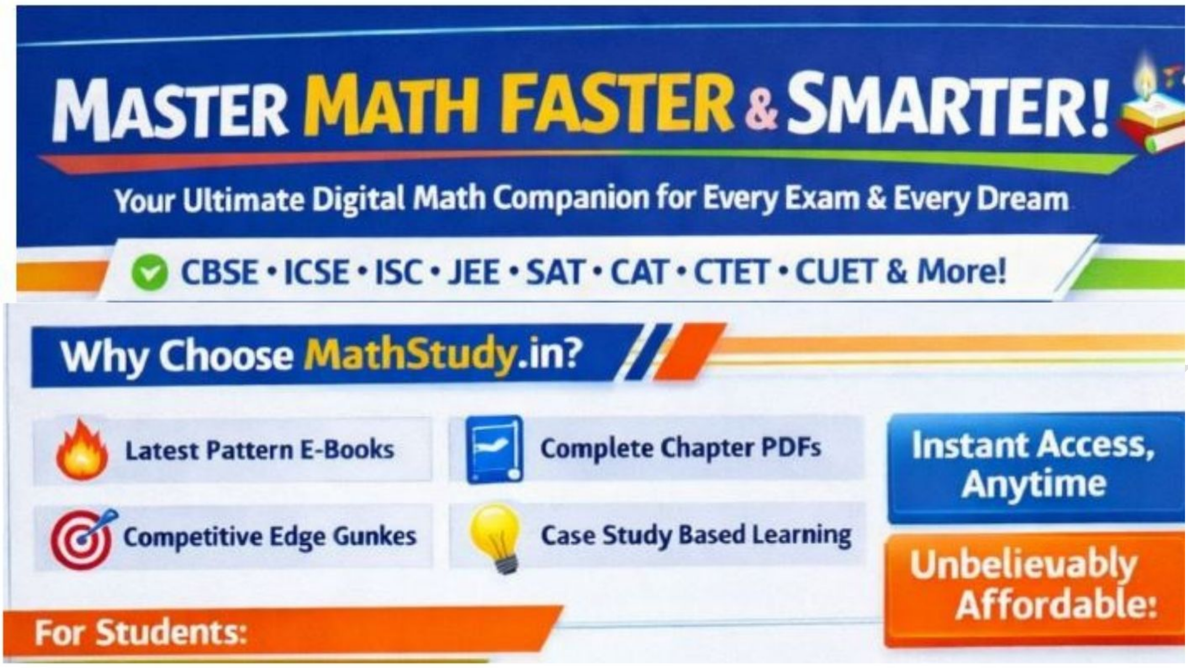

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Solutions

1. **Correct Option: (C)**. Marginal Revenue $MR = \frac{dR}{dx} = 26x + 26$. At $x = 7$, $MR = 26(7) + 26 = 182 + 26 = 208$.

2. **Correct Option: (A)**. $V = \pi r^2 h = \pi(10)^2 h = 100\pi h$. $\frac{dV}{dt} = 100\pi \frac{dh}{dt}$. Given $\frac{dV}{dt} = 314 \approx 100\pi$ (since $\pi \approx 3.14$). So $314 = 100(3.14)\frac{dh}{dt} \implies \frac{dh}{dt} = 1$ m/h.

3. **Correct Option: (A)**. $f'(x) = 2x - 4$. For strictly decreasing, $2x - 4 < 0 \implies x < 2$. Thus, $x \in (-\infty, 2)$.

4. **Correct Option: (B)**. Differentiating $x^2 + y^2 = 25$ gives $2x + 2yy' = 0 \implies y' = -x/y$. Slope of tangent at $(3, 4)$ is $-3/4$. Slope of normal is $-1/y' = 4/3$.

5. **Correct Option: (D)**. $y' = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2 - x)$. For increasing, $y' > 0$. Since $e^{-x} > 0$, we need $x(2 - x) > 0 \implies x \in (0, 2)$.

6. **Correct Option: (B)**. Let $y = (1/x)^x \implies \log y = x \log(1/x) = -x \log x$. $\frac{1}{y}y' = -[x(1/x) + \log x] = -(1 + \log x)$. $y' = 0$ at $\log x = -1 \implies x = 1/e$. Max value is $(e)^{1/e}$.

7. **Correct Option: (A)**. $x^2 + y^2 = 25 \implies 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$. Given $x = 4$, $\frac{dx}{dt} = 2$. Since $x = 4, y = 3$. $2(4)(2) + 2(3)\frac{dy}{dt} = 0 \implies 16 + 6\frac{dy}{dt} = 0 \implies \frac{dy}{dt} = -8/3$. Speed is $8/3$ cm/s.

8. **Correct Option: (A)**. $y = x^2 \implies \frac{dy}{dx} = 2x$. Tangent at $45^\circ \implies$ slope $= \tan 45^\circ = 1$. $2x = 1 \implies x = 1/2$. Then $y = (1/2)^2 = 1/4$.

9. **Correct Option: (B)**. $f'(x) = \cos x + \sin x$. For increasing, $\cos x + \sin x > 0 \implies \sqrt{2}\sin(x + \pi/4) > 0$. This holds for $x + \pi/4 \in (0, \pi) \implies x \in (-\pi/4, 3\pi/4)$.

10. **Correct Option: (A)**. $V = \frac{4}{3}\pi r^3 \implies \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$. $4.5 = 4\pi(3)^2\frac{dr}{dt} \implies 4.5 = 36\pi\frac{dr}{dt} \implies \frac{dr}{dt} = \frac{4.5}{36\pi} = \frac{1}{8\pi}$.

11. **Correct Option: (D)**. $y' = 4x + 3\cos x$. At $x = 0$, $y' = 3(1) = 3$. Slope of normal $= -1/3$.

12. **Correct Option: (D)**. (A) $\sin 2x \implies 2\cos 2x$ (positive in first half); (B) $\tan x \implies \sec^2 x$ (always positive); (C) $\cos 3x \implies -3\sin 3x$ (changes sign); (D) $\cos x \implies -\sin x$. On $(0, \pi/2)$, $\sin x$ is positive, so $-\sin x$ is negative.

13. **Correct Option: (B)**. Rectangle with max area in circle is a square. Diagonal $= 2r$, so side $s = \frac{2r}{\sqrt{2}} = \sqrt{2}r$. Area $= s^2 = 2r^2$.

14. **Correct Option: (A)**. $V = \frac{4}{3}\pi r^3, S = 4\pi r^2$. $\frac{dV}{dS} = \frac{dV/dr}{dS/dr} = \frac{4\pi r^2}{8\pi r} = r/2$. At $r = 2$, ratio is 1.

15. **Correct Option: (A)**. Line $2y + x - 7 = 0$ has slope $-1/2$. Tangent perpendicular to it must have slope $m = 2$. $y' = 2x - 4 = 2 \implies 2x = 6 \implies x = 3$. $y = 9 - 12 + 5 = 2$. Point $(3, 2)$.

16. **Correct Option: (A)**. $f'(x) = 3x^2 - 3 = 3(x - 1)(x + 1)$. $f''(x) = 6x$. $f''(-1) = -6 < 0$ (Max). $f(-1) = -1 + 3 = 2$.

17. **Correct Option: (B)**. $f'(x) = 3x^2 - 36x + 96 = 3(x - 4)(x - 8)$. Critical points: 4, 8. $f(0) = 0$, $f(4) = 64 - 18(16) + 96(4) = 160$, $f(8) = 512 - 18(64) + 96(8) = 128$, $f(9) = 135$. Minimum is 0 at $x = 0$.

18. **Correct Option: (B)**. $x = 60 - y$. $P = (60 - y)y^3 = 60y^3 - y^4$. $P' = 180y^2 - 4y^3 = 0 \implies 4y^2(45 - y) = 0 \implies y = 45$. Then $x = 60 - 45 = 15$.

19. **Correct Option: (C)**. $f'(x) = \frac{\log x - x(1/x)}{(\log x)^2} = \frac{\log x - 1}{(\log x)^2}$. Increasing if $\log x - 1 > 0 \implies \log x > 1 \implies x > e$.

20. **Correct Option: (A)**. At $x = 0, y = 1$. Point $(0, 1)$. $y' = 2e^{2x} + 2x \implies$ at $x = 0, m = 2$. Tangent: $y - 1 = 2(x - 0) \implies 2x - y + 1 = 0$. Distance from $(0, 0) = \frac{|2(0) - 0 + 1|}{\sqrt{2^2 + (-1)^2}} = 1/\sqrt{5}$.