PRACTICE QUESTION PAPER - II

CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each.
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs)

1. If the function $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = x^2 + 1$, then $f^{-1}(17)$ is equal to:
   (a) 4
   (b) $\{-4, 4\}$
   (c) 3
   (d) $\{3, -3\}$

2. Let $A = \{1, 2, 3\}$. The total number of non-empty subsets of A is:
   (a) 6
   (b) 7
   (c) 8
   (d) 9

3. The value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ is:
   (a) π
   (b) 2π
   (c) $\frac{3\pi}{4}$

(d) $\frac{5\pi}{4}$

4. If $f(x) = \frac{x}{x-1}, x \neq 1$, then $f(f(x))$ is equal to:

(a) x

(b) $\frac{1}{x}$

(c) x^2

(d) $\frac{x}{(x-1)^2}$

5. The range of $\sec^{-1} x$ is:

(a) $[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$

(b) $[0, \pi] - \{\frac{\pi}{2}\}$

(c) $(-\infty, \infty)$

(d) $[-\pi, \pi]$

6. If $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, then A^4 is equal to:

(a) $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$

(c) $\begin{bmatrix} 16 & 16 \\ 16 & 16 \end{bmatrix}$

(d) $\begin{bmatrix} 2^4 & 2^4 \\ 2^4 & 2^4 \end{bmatrix}$

7. If A is a skew-symmetric matrix, then A^T is equal to:

(a) A

(b) $-A$

(c) I

(d) 0

8. If A is a matrix of order 3×4, then each row of A has:

(a) 3 elements

(b) 4 elements

(c) 7 elements

(d) 12 elements

9. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then the value of x is:

(a) 3

(b) 4

(c) ± 6

(d) ± 3

10. If A is a non-singular matrix, then $adj(adj(A))$ is equal to:

(a) $|A|A$
(b) $|A|^2A$
(c) $|A|^{n-2}A$ (where n is order)
(d) $|A|^3A$

11. The function $f(x) = \sin x + \cos x$ is increasing in the interval:

(a) $(0, \frac{\pi}{2})$
(b) $(\frac{\pi}{4}, \frac{\pi}{2})$
(c) $(0, \frac{\pi}{4})$
(d) $(\frac{\pi}{2}, \pi)$

12. The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is:

(a) $e^y = e^x + \frac{x^3}{3} + C$
(b) $e^y = e^x + \frac{x^3}{3}$
(c) $e^{-y} = e^x + \frac{x^3}{3} + C$
(d) $e^y = e^x + x^3 + C$

13. If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is equal to:

(a) $25y$
(b) $5y$
(c) $-25y$
(d) $10y$

14. The value of $\int_1^e \log x\, dx$ is:

(a) $e - 1$
(b) 1
(c) 0
(d) $\frac{1}{e}$

15. The area bounded by $y = |x|$ and $y = 1$ is:

(a) 1 sq. unit
(b) 2 sq. units
(c) $\frac{1}{2}$ sq. unit
(d) 4 sq. units

16. If $\mathbf{r} = \begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$, then $\frac{\partial \mathbf{r}}{\partial z}$ is:

(a) $4x + y - 2z$
(b) -3
(c) -3
(d) $5 - 8$

17. The angle between the vector $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and the plane $\mathbf{r} \cdot (\hat{i} + \hat{k}) = 1$ is:

   (a) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
   (b) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
   (c) $\sin^{-1}\left(\frac{1}{3}\right)$
   (d) 0

18. The point of intersection of the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z+2}{3}$ and the xy-plane is:

   (a) $\left(\frac{2}{3}, \frac{1}{3}, 0\right)$
   (b) $(0, 1, -2)$
   (c) $\left(\frac{1}{3}, \frac{2}{3}, 0\right)$
   (d) $\left(-\frac{2}{3}, \frac{1}{3}, 0\right)$

Assertion-Reasoning Based Questions

Questions 19 and 20 are Assertion-Reasoning based questions. In these questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** The maximum value of $\frac{\log x}{x}$ is $\frac{1}{e}$. **Reason (R):** $\frac{d}{dx}\left(\frac{\log x}{x}\right) = \frac{1-\log x}{x^2}$.

20. **Assertion (A):** The binary operation $*$ defined on $\mathbb{Z}$ by $a * b = ab + 1$ is commutative. **Reason (R):** An operation $*$ is commutative if $a * b = b * a$ for all $a, b \in \mathbb{Z}$.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Find $\frac{dy}{dx}$, if $y = (\cos x)^x + x^{\sin x}$.

22. Given $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$. Show that $A^2 - 6A + 11I = 0$, where I is the identity matrix.

23. Find the magnitude of $\vec{a} \times \vec{b}$, if $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$.

OR

Find the area of the parallelogram whose adjacent sides are represented by the vectors

$$\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$$

and

$$\vec{b} = \hat{i} - 4\hat{j} + 2\hat{k}.$$

24. Find the interval in which the function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is strictly decreasing.

25. A bag contains 5 red and 3 black balls. If three balls are drawn at random without replacement, find the probability that exactly two balls are red.

OR

If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, find $P(A|B)$.

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

27. Evaluate $\int \frac{(x^2+1)e^x}{(x+1)^2}\, dx$.

28. Find the coordinates of the point where the line $\frac{x+2}{1} = \frac{y-5}{3} = \frac{z+1}{-2}$ intersects the xz-plane.

OR

Find the angle between the lines $\vec{r} = 3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$ and $\vec{r} = 5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$.

29. Find the general solution of the differential equation $\frac{dy}{dx} + 2y\tan x = \sin x$.

OR

Show that the differential equation $(x-y)\frac{dy}{dx} = x + 2y$ is homogeneous and solve it.

30. Express the matrix $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

31. Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{2}{11}\right) = \tan^{-1}\left(\frac{3}{4}\right)$.

32. Solve the following Linear Programming Problem graphically: Minimize $Z = 3x + 5y$ subject to the constraints $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.

OR

A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, Automatic and Hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machine to manufacture a packet of screw A, while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screw B. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screw A at a profit of $Rs.7$ and screw B at a profit of $Rs.10$. Assuming that he can sell all the screws he manufactures, how many packets of each type should be made in a day to maximize his profit? (Formulate as LPP).

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

34. Using the method of integration, find the area of the triangular region whose vertices are $(1,0)$, $(2,2)$, and $(3,1)$.

35. Evaluate $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}\,dx$.

OR

Find the derivative of $\sin x$ with respect to x^x.

36. Find the equation of the plane passing through the point $(-1,3,2)$ and perpendicular to each of the planes $x+2y+3z=5$ and $3x+3y+z=0$.

OR

Find the value of p so that the lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are perpendicular to each other.

37. A wire of length 36 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of the two pieces so that the combined area of the square and the circle is minimum?

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

39. **Case Study 1: Production Cost and Marginal Cost**

A company manufactures electronic devices. The cost function for manufacturing x units of a product is given by $C(x)=500+10x+0.005x^2$. The Marginal Cost (MC) is the instantaneous rate of change of total cost $C(x)$ with respect to the number of units produced x.

Based on the given information, answer the following questions:

(a) Find the Marginal Cost function $MC(x)$. (1 Mark)

(b) Calculate the Marginal Cost when 100 units are produced. (1 Mark)

(c) Determine the number of units x for which the Average Cost is minimum. (2 Marks)

OR

(d) Find the number of units x at which the Marginal Cost is equal to the Average Cost. (2 Marks)

40. **Case Study 2: Bridge Construction and Skew Lines**

An engineer is planning the design for a cable-stayed bridge. Two main cable segments are modeled as two lines L_1 and L_2 in space. Line L_1 passes through $P(3,1,5)$ and is parallel to the vector $\vec{b}_1=\hat{i}+\hat{j}-\hat{k}$. Line L_2 passes through $Q(4,-2,3)$ and is parallel to the vector $\vec{b}_2=2\hat{i}+\hat{j}+3\hat{k}$.

Based on the given information, answer the following questions:

(a) Write the vector equation of line L_1. (1 Mark)

(b) Show that the lines L_1 and L_2 are skew lines. (1 Mark)

(c) Find the dot product of the direction vectors $\vec{b}_1$ and $\vec{b}_2$. (2 Marks)

OR

(d) Find the vector perpendicular to both lines L_1 and L_2. (2 Marks)

41. **Case Study 3: Bernoulli Trials and Binomial Distribution**

In a game of archery, the probability of a player hitting the target is $\frac{1}{4}$. The player is allowed to shoot 5 arrows independently.

Based on the given information, answer the following questions:

(a) What is the probability of the player hitting the target exactly 3 times? (1 Mark)

(b) Find the probability of the player hitting the target at least once. (2 Marks)

OR

(c) If the player shoots n arrows, and the probability of hitting the target at least once is greater than $\frac{1}{2}$, find the minimum value of n. (2 Marks)