SOLUTIONS

www.udgamwelfarefoundation.com

For Best Mathematics E-Books, Visit:
www.mathstudy.in

SOLUTIONS: AREAS OF PARALLELOGRAMS AND TRIANGLES

Mathematics | Class IX (2026/SOL-ARPARA/09/001)

Section A: Multiple Choice Questions

1. **(b) 1 : 2**
   Reason: Area of triangle = $\frac{1}{2}$ × base × height, Area of parallelogram = base × height.

2. **(c)** $30\,cm^2$
   Reason: The sum of areas of triangles formed by a point on the opposite side is half the area of the parallelogram.

3. **(a) 1 : 1**
   Reason: Areas of parallelograms on equal bases and between the same parallels are equal.

4. **(b)** $4\,cm^2$
   Reason: Joining mid-points divides the triangle into 4 triangles of equal area. Area(DEF) $= \frac{1}{4} \times 16 = 4\,cm^2$.

5. **(c) Remains same**
   Reason: New Area = (2 × base) × ($\frac{1}{2}$ × height) = base × height.

6. **(d) Triangles of equal area**
   Reason: Median divides a triangle into two triangles of equal area.

7. **(a)** $\frac{1}{2}area(PQRS)$
   Reason: Property of any point inside a parallelogram regarding triangles on opposite bases.

8. **(a) 1 : 1**
   Reason: Both are parallelograms sharing the same base and parallels.

Section B: Very Short Answer Questions

1. **Proof:** $\triangle DAC$ and $\triangle EAC$ are on the same base AC and between same parallels $AC \parallel DE$. $\therefore area(\triangle DAC) = area(\triangle EAC)$. Adding $area(\triangle ABC)$ to both sides: $area(\triangle DAC) + area(\triangle ABC) = area(\triangle EAC) + area(\triangle ABC) \implies area(ABCD) = area(\triangle ABE)$.

2. **Proof:** Diagonal AC of $\|^{gm}$ $ABCD$ forms $\triangle ABC$ and $\triangle ADC$. In these triangles, $AB = CD$, $BC = AD$, and AC is common. By SSS congruence, $\triangle ABC \cong \triangle ADC$. Congruent triangles have equal areas.

3. **Proof:** AD is median $\implies area(\triangle ABD) = \frac{1}{2}area(\triangle ABC)$. In $\triangle ABD$, BE is median (since E is mid-point of AD). $\therefore area(\triangle BED) = \frac{1}{2}area(\triangle ABD) = \frac{1}{2}(\frac{1}{2}area(\triangle ABC)) = \frac{1}{4}area(\triangle ABC)$.

4. **Proof:** Join HF. $ABFH$ and $HFCD$ are parallelograms. $Area(\triangle HGF) = \frac{1}{2}Area(HFCD)$ and $Area(\triangle HEF) = \frac{1}{2}Area(ABFH)$. Summing these gives $area(EFGH) = \frac{1}{2}area(ABCD)$.

Section C: Short Answer Questions

1. **Proof:** Join CD. $Area(\triangle BCD) = \frac{1}{2}Area(\triangle ABC)$ (since D is mid-point). $\triangle PDQ$ and $\triangle PDC$ are on same base PD and between $PD \parallel CQ$. $\therefore area(\triangle PDQ) = area(\triangle PDC)$. $Area(\triangle BPQ) = area(\triangle BPD) + area(\triangle PDQ) = area(\triangle BPD) + area(\triangle PDC) = area(\triangle BCD) = \frac{1}{2}area(\triangle ABC)$.

2. **Proof:** (i) Parallelograms $PQRS$ and $ABRS$ are on the same base SR and between same parallels $SR \parallel PB$. Hence, $area(PQRS) = area(ABRS)$. (ii) $\triangle AXS$ and $\parallel^{gm}$ $ABRS$ are on the same base AS and between $AS \parallel BR$. $\therefore area(\triangle AXS) = \frac{1}{2}area(ABRS) = \frac{1}{2}area(PQRS)$.

3. **Proof:** $area(\triangle AOD) = area(\triangle BOC)$. Adding $area(\triangle DOC)$ to both sides: $area(\triangle AOD) + area(\triangle DOC) = area(\triangle BOC) + area(\triangle DOC) \implies area(\triangle ADC) = area(\triangle BDC)$. Since these triangles have equal areas and share the same base DC, they must lie between the same parallels. Thus $AB \parallel DC$, making $ABCD$ a trapezium.

Section D: Long Answer Questions

1. **Proof:** $\triangle ABE$ and parallelogram $EBCY$ (where Y is intersection) share same base BE and parallels? No. Note: $area(\triangle ABE) = area(\triangle CBE)$ (Same base BE and $BE \parallel AC$). Also, $\triangle ACF$ and $\triangle BCF$ are on same base CF and $CF \parallel AB$. $\triangle BCF$ and $\triangle CBE$ are on same base BC and $BC \parallel EF$. $\therefore area(\triangle CBE) = area(\triangle BCF)$. Thus, $area(\triangle ABE) = area(\triangle ACF)$.

2. **Proof:** Let AB be the base. For equal areas, the altitude h of the parallelogram must equal the side AF of the rectangle (since $Area_{rect} = AB \times AF$ and $Area_{para} = AB \times h$). In the parallelogram, let AD be the other side. The altitude h is the perpendicular distance, while AD is the slant side (hypotenuse). In a right triangle, hypotenuse $>$ altitude ($AD > h$). Perimeter of Rectangle $= 2(AB + AF)$. Perimeter of Parallelogram $= 2(AB + AD)$. Since $AD > h$ and $h = AF$, then $AD > AF$. $\therefore 2(AB + AD) > 2(AB + AF)$.

Section E: Case Study Based Questions

1. **(c)** $600\,m^2$ (PM is median, so half area).
2. **(b)** $400\,m^2$ ($Area = \frac{1}{3}Area(\triangle PQR)$ for centroid/ratio $2:1$).
3. **(a)** $area(\triangle POQ) = area(\triangle POR)$ (By symmetry/ratio property).
4. **(b)** $Rs.20,000$ ($Area(POQ) = 400\,m^2$. Cost $= 400 \times 50 = 20,000$).
5. **(b)** **1 : 3** ($400 : 1200 = 1 : 3$).