Chapter 9: Areas of Parallelograms and Triangles

This chapter explores how to calculate areas of parallelograms and triangles, and the important relationships between their areas when they share common bases and heights.

—

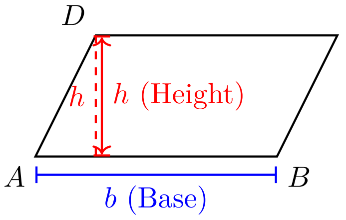
1. Basic Area Formulas

These are the fundamental formulas for calculating areas.

1. **Area of a Parallelogram**

$$A = b \times h$$

- A: Area of the parallelogram.
- b: Length of any base of the parallelogram.
- h: Perpendicular height (distance between the base and its opposite side).
- **Usage**: Multiply the base length by the perpendicular height. Make sure the height is measured perpendicular to the base, not along the slanted side.

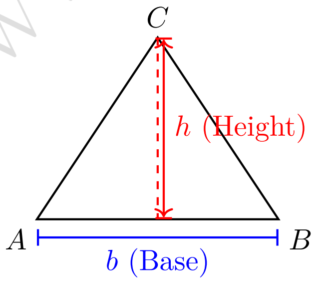


Area = Base × Height

2. **Area of a Triangle**

$$A = \frac{1}{2} \times b \times h$$

- A: Area of the triangle.
- b: Length of any side (base).
- h: Perpendicular height from the opposite vertex to that base.
- **Usage**: Multiply base by height, then take half. Any side can be the base, but use the corresponding height.

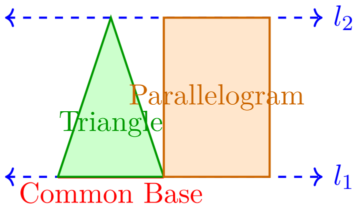


Area = $\frac{1}{2}$ × Base × Height

—

2. Figures on the Same Base and Between Same Parallels

This is the core concept of the chapter. Two figures are said to be on the same base and between the same parallels if: 1. They share a common base. 2. Their opposite vertices (or sides) lie on a line parallel to the base.



Parallel lines

—

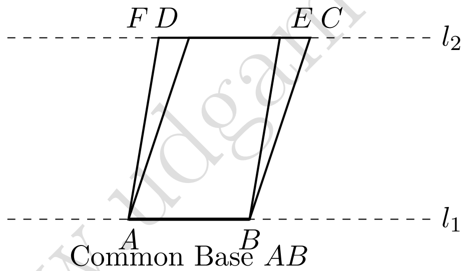
3. Important Theorems about Area Relationships

These theorems establish relationships between areas of different shapes sharing common bases and heights.

3. **Parallelograms on the Same Base and Between Same Parallels**

$$\text{Area}(ABCD) = \text{Area}(ABEF)$$

- **Meaning**: Parallelograms on the same base and between the same parallels have equal areas.
- **Condition**: Both parallelograms must share base AB and be between parallels l_1 and l_2.
- **Usage**: To prove areas are equal without calculation.



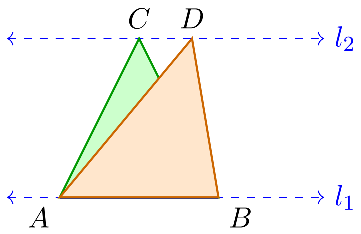
Common Base AB

$\text{Area}(ABCD) = \text{Area}(ABEF)$

4. **Triangles on the Same Base and Between Same Parallels**

$$\text{Area}(\triangle ABC) = \text{Area}(\triangle ABD)$$

- **Meaning**: Triangles on the same base and between the same parallels have equal areas.
- **Condition**: Both triangles must share base AB and have their third vertices on line l_2 parallel to AB.
- **Usage**: To compare areas of triangles without calculating.

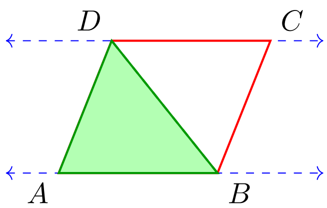


Common Base AB
Area($\triangle ABC$) = Area($\triangle ABD$)

5. **Relationship Between Area of Parallelogram and Triangle**

$$\text{Area of parallelogram } ABCD = 2 \times \text{Area of } \triangle ABD$$

- **Condition**: When a parallelogram and a triangle are on the same base and between the same parallels.
- **Alternative**: Area of $\triangle ABD = \frac{1}{2} \times$ Area of parallelogram $ABCD$
- **Usage**: To find one area when the other is known.

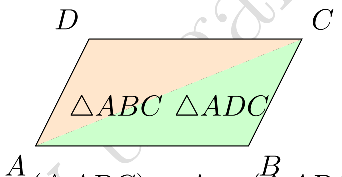


Area($ABCD$) = 2 × Area($\triangle ABD$)

6. **Diagonals of a Parallelogram Divide it into Equal Triangles**

$$\text{Area}(\triangle ABC) = \text{Area}(\triangle ADC)$$

- **Meaning**: A diagonal of a parallelogram divides it into two triangles of equal area.
- **Usage**: Useful for solving problems where a parallelogram is divided by its diagonal.



Area($\triangle ABC$) = Area($\triangle ADC$)

—

4. Median of a Triangle and Area

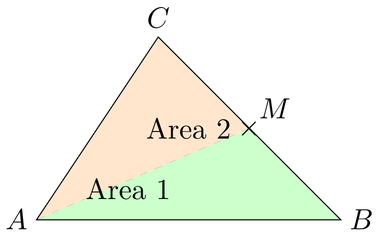
A median connects a vertex to the midpoint of the opposite side.

7. **Median Divides Triangle into Two Equal Areas**

$$\text{Area}(\triangle ABM) = \text{Area}(\triangle ACM) = \frac{1}{2}\text{Area}(\triangle ABC)$$

- M: Midpoint of side BC.

- AM: Median from vertex A.
- **Meaning**: A median of a triangle divides it into two triangles of equal area.
- **Usage**: Useful when a triangle is divided by its median.



Area 1 = Area 2 = $\frac{1}{2}$ Area($\triangle ABC$)

—

Quick Revision Summary

Here are all the essential formulas and relationships from this chapter.

1. **Area of Parallelogram**: $A = b \times h$
2. **Area of Triangle**: $A = \frac{1}{2} \times b \times h$
3. **Parallelograms on same base, same parallels**: Equal areas
4. **Triangles on same base, same parallels**: Equal areas
5. **Parallelogram and Triangle relationship**: Area(parallelogram) = 2 × Area(triangle) on same base between same parallels
6. **Diagonal of parallelogram**: Divides it into two triangles of equal area
7. **Median of triangle**: Divides it into two triangles of equal area: Area($\triangle ABM$) = Area($\triangle ACM$) = $\frac{1}{2}$Area($\triangle ABC$)