Case Study 1

In the modern digital era, the amount of data generated daily is staggering. A high-tech data center in Bangalore manages vast amounts of information stored in binary format. Information is measured in bits and bytes, where 1 byte consists of 2^3 bits. The storage capacity of a single server rack is approximately 2^{40} bytes. During a peak hour, the data center processes 10^9 transactions per second. Engineers often use exponents to simplify these massive numbers and perform quick calculations using the laws of exponents. For instance, when doubling the storage capacity, they simply increase the power of 2 by one. Furthermore, the size of a single transistor on a microchip within these servers is incredibly small, measuring about 0.000000007 meters, which is written in standard form as 7×10^{-9} meters. Understanding how to multiply, divide, and simplify these powers is crucial for software engineers to ensure the system does not crash under heavy loads. If the data center expands its capacity by a factor of 2^5, the total storage becomes a product of the initial capacity and the expansion factor, demonstrating the practical application of the product law of exponents.

Questions

1. A single server rack has 2^{40} bytes of storage. If 1 byte equals 2^3 bits, what is the total storage capacity of the rack in bits?

   (a) 2^{37} bits
   (b) 2^{43} bits
   (c) 2^{120} bits
   (d) 4^{43} bits

   Answer: (b) 2^{43} bits
   Solution: Using the law of exponents $a^m \times a^n = a^{m+n}$, the total bits $= 2^{40} \times 2^3 = 2^{40+3} = 2^{43}$ bits.

2. If the data center increases its current storage of 2^{40} bytes by a factor of 2^5, what is the new storage capacity?

   (a) 2^8 bytes
   (b) 2^{45} bytes
   (c) 2^{200} bytes
   (d) 4^{45} bytes

   Answer: (b) 2^{45} bytes
   Solution: Expansion means multiplication. New capacity $= 2^{40} \times 2^5 = 2^{40+5} = 2^{45}$ bytes.

3. The size of a transistor is 7×10^{-9} meters. If a newer model reduces this size by dividing it by 10^{-2}, what is the new size in meters?

   (a) 7×10^{-11} meters
   (b) 7×10^{-7} meters
   (c) 7×10^{-18} meters
   (d) 0.7×10^{-9} meters

   Answer: (b) 7×10^{-7} meters
   Solution: Using the quotient law $a^m \nabla \cdot a^n = a^{m-n}$, the new size $= \frac{7 \times 10^{-9}}{10^{-2}} = 7 \times 10^{-9-(-2)} = 7 \times 10^{-9+2} = 7 \times 10^{-7}$ meters.

4. A backup system stores half of the storage of a server rack. If the rack capacity is 2^{40} bytes, how is the backup storage represented in exponential form?

   (a) 1^{40} bytes

   (b) 2^{20} bytes

   (c) 2^{39} bytes

   (d) 2^{41} bytes

   Answer: (c) 2^{39} bytes
   Solution: Half of a value means dividing by 2 (which is 2^1). So, $2^{40} \nabla \cdot 2^1 = 2^{40-1} = 2^{39}$ bytes.

5. Simplify the expression representing the ratio of a server's peak transactions (10^9) to its idle transactions (10^4).

   (a) 10^{13}

   (b) 10^{36}

   (c) 10^5

   (d) 1^5

   Answer: (c) 10^5
   Solution: Ratio means division. $\frac{10^9}{10^4} = 10^{9-4} = 10^5$.