Case Study 2

Space scientists use very large numbers to describe the distances between planets and the brightness of stars. Our Sun is about 1.5×10^8 kilometers away from Earth. To make these numbers easier to read, scientists use exponents. For example, the mass of a small moon might be written as $3^4 \times 3^5$ kilograms. When scientists launch a rocket, they must calculate the force needed. If a rocket travels at a speed of 10^4 meters per second and continues for 10^3 seconds, the distance covered is found by multiplying these powers. Sometimes, they need to find the square of a distance, like $(5^3)^2$, which uses the power of a power law. On the other hand, the size of a tiny space dust particle might be as small as 0.00005 meters. Using the laws of exponents helps these scientists avoid making mistakes with many zeros. By using rules like the product law and the quotient law, they can quickly find out how much larger one planet is compared to another or how long it will take to reach a distant star.

Questions

1. A moon has a mass represented by $3^4 \times 3^5$ kilograms. What is the simplified mass of the moon in exponential form?

   (a) 3^{20} kg
   (b) 3^9 kg
   (c) 9^9 kg
   (d) 3^1 kg

   Answer: (b) 3^9 kg
   Solution: Using the product law $a^m \times a^n = a^{m+n}$, we add the powers: $3^{4+5} = 3^9$ kg.

2. A rocket covers a distance of $(5^3)^2$ kilometers. What is this distance simplified?

   (a) 5^5 km
   (b) 5^6 km
   (c) 10^3 km
   (d) 5^9 km

   Answer: (b) 5^6 km
   Solution: Using the power of a power law $(a^m)^n = a^{m \times n}$, we multiply the powers: $5^{3 \times 2} = 5^6$ km.

3. The distance to a planet is 10^{12} meters. If a probe has already traveled 10^8 meters, how many times more distance does it need to cover?

   (a) 10^{20} times
   (b) 10^4 times
   (c) $10^{1.5}$ times
   (d) 10^0 times

   Answer: (b) 10^4 times
   Solution: To find "how many times," we divide the total distance by the distance traveled: $10^{12} \nabla \cdot 10^8 = 10^{12-8} = 10^4$.

4. What is the value of $(2^3 \times 5^3)$ in a simpler exponential form?

   (a) 10^3

(b) 10^6

(c) 10^9

(d) 7^3

Answer: (a) 10^3
Solution: Using the law $a^m \times b^m = (ab)^m$, we get $(2 \times 5)^3 = 10^3$.

5. A scientist finds that a star's brightness is 7^{15} units. If the brightness is reduced by 7^{15} times due to space dust, what is the remaining brightness?

(a) 7^1 units

(b) 7^0 units (which is 1)

(c) 7^{30} units

(d) 0 units

Answer: (b) 7^0 units
Solution: Reducing by a factor means division: $7^{15} \nabla \cdot 7^{15} = 7^{15-15} = 7^0$. Any non-zero number raised to the power 0 is 1.