# Case Study 3

In a school science lab, students are studying how bacteria grow. Bacteria are tiny living things that can only be seen under a microscope. Some bacteria can double their number every hour. If a scientist starts with $2^1$ bacterium, after one hour there are $2^2$, after two hours there are $2^3$, and so on. This is called exponential growth. The students also learned about the size of different cells. A plant cell might be $10^{-4}$ meters long, while a very small virus might be $10^{-8}$ meters long. To compare these sizes, students use the laws of exponents. For example, to find out how many times larger a cell is than a virus, they divide the powers. They also use rules to simplify calculations, like when they need to find the total area of a rectangular slide by multiplying its length and width, both written in powers. Using exponents makes it much easier to handle these very small measurements without getting confused by many decimal places.

## Questions

1. If the number of bacteria in a dish doubles every hour and starts at $2^5$, what will the number be after $2^3$ more doublings?

   (a) $2^{15}$

   (b) $2^8$

   (c) $2^2$

   (d) $4^8$

   **Answer:** (b) $2^8$
   **Solution:** Using the product law $a^m \times a^n = a^{m+n}$, we have $2^5 \times 2^3 = 2^{5+3} = 2^8$.

2. A microscope zooms in on a cell. The zoom level is written as $(10^2)^3$. What is this zoom level in a single power of 10?

   (a) $10^5$

   (b) $10^8$

   (c) $10^6$

   (d) $10^{23}$

   **Answer:** (c) $10^6$
   **Solution:** Using the power of a power law $(a^m)^n = a^{m \times n}$, we get $10^{2 \times 3} = 10^6$.

3. A plant cell is $10^{-4}$ meters and a virus is $10^{-8}$ meters. How many viruses could fit along the length of one plant cell?

   (a) $10^{-12}$

   (b) $10^4$

   (c) $10^{-4}$

   (d) $10^{12}$

   **Answer:** (b) $10^4$
   **Solution:** We divide the larger size by the smaller size: $10^{-4} \nabla \cdot 10^{-8} = 10^{-4-(-8)} = 10^{-4+8} = 10^4$.

4. Simplify the expression for the total number of nutrients in a sample if it is $(2 \times 3)^4$ units.

   (a) $2^4 \times 3^4$

   (b) $2^4 + 3^4$

(c) $6^8$

(d) $5^4$

**Answer:** (a) $2^4 \times 3^4$

**Solution:** According to the law $(ab)^m = a^m \times b^m$, the expression $(2 \times 3)^4$ is equal to $2^4 \times 3^4$.

5. If a scientist has $5^6$ samples and gives away $5^6$ samples for testing, how many samples are left in exponential form?

   (a) $5^1$

   (b) 0

   (c) $5^0$

   (d) $1^6$

**Answer:** (c) $5^0$

**Solution:** $5^6 \nabla \cdot 5^6 = 5^{6-6} = 5^0$. Since $5^0 = 1$, this represents the one remaining group or the mathematical identity for the division.