Case Study 1

The Green Club of Silver Oaks International School has decided to develop a rectangular community garden to grow organic vegetables. To manage the space efficiently, the students divided the garden into different sections. The length of the tomato patch is represented by $3x + 4$ meters, while the width is $2x - 1$ meters. In another section, the students are planting carrots in a row that is $5x - 2$ meters long.

To decorate the perimeter of the entire garden, the students need to calculate the total boundary length using algebraic expressions. During the planning phase, the club treasurer, Rohan, noted that the cost of seeds for each square meter is represented by the expression $4x^2 + 7x - 5$, where x represents a variable related to the soil quality index. Furthermore, the club received a donation of $10x + 15$ packets of organic fertilizer but had to return $3x - 4$ damaged packets to the supplier. The students must now simplify these expressions to determine the actual amount of materials needed, identify the constants and coefficients to manage their budget, and ensure that the addition and subtraction of these expressions are handled accurately before the planting season begins next week.

Questions

1. If the length of the tomato patch is $3x + 4$ and the width is $2x - 1$, which of the following expressions represents the perimeter of this rectangular patch?

   (a) $5x + 3$

   (b) $10x + 6$

   (c) $6x^2 + 5x - 4$

   (d) $5x - 3$

   Answer: (b) $10x + 6$
   Solution: The perimeter P of a rectangle is given by the formula $P = 2(L + W)$. Here, $L = 3x + 4$ and $W = 2x - 1$. $P = 2((3x + 4) + (2x - 1)) = 2(3x + 2x + 4 - 1) = 2(5x + 3) = 10x + 6$.

2. In the expression for the cost of seeds, $4x^2 + 7x - 5$, what is the coefficient of x and the constant term respectively?

   (a) 4 and 5

   (b) 7 and 5

   (c) 7 and -5

   (d) 4 and -5

   Answer: (c) 7 and -5
   Solution: In the expression $4x^2 + 7x - 5$, the term with x is $7x$, so the coefficient is 7. The term without any variable is -5, which is the constant.

3. The club had $10x + 15$ packets of fertilizer and returned $3x - 4$ packets. How many packets of fertilizer are left with the club?

   (a) $7x + 11$

   (b) $13x + 11$

   (c) $7x + 19$

   (d) $13x + 19$

Answer: (c) $7x + 19$
Solution: Packets left = (Initial packets) - (Returned packets)
$= (10x + 15) - (3x - 4) = 10x + 15 - 3x + 4$
Grouping like terms: $(10x - 3x) + (15 + 4) = 7x + 19$.

4. Which of the following sets contains only "like terms" based on the expressions mentioned in the case study?

   (a) $\{3x, 2x, 4x^2\}$
   (b) $\{4, -1, -5\}$
   (c) $\{7x, 10x, 15\}$
   (d) $\{3x, 5x, 4x^2\}$

   Answer: (b) $\{4, -1, -5\}$
   Solution: Like terms must have the same variable and the same power. 4, -1, and -5 are all constant terms (no variables), making them like terms. Option (a), (c), and (d) include terms with different variables or powers (x vs x^2 or constants).

5. If the students want to combine the length of the tomato patch $(3x + 4)$ and the carrot row $(5x - 2)$ to find a total linear distance, what is the simplified expression?

   (a) $8x + 6$
   (b) $8x + 2$
   (c) $2x + 6$
   (d) $15x - 8$

   Answer: (b) $8x + 2$
   Solution: Total distance = $(3x + 4) + (5x - 2)$
   $= 3x + 5x + 4 - 2 = 8x + 2$.