Case Study 1: Designing a Ramp using Trigonometric Ratios

A group of architecture students is designing a wheelchair ramp for a public library. According to safety guidelines, the ramp should make a gentle incline with the ground. The students decide to test their design by first building a small model. In the model, the ramp forms a right-angled triangle with the ground and the vertical wall. They measure the base and height of the ramp in their scaled model and then use trigonometric ratios to find the angle of inclination of the ramp. They also verify certain trigonometric identities to check their understanding and ensure accuracy in calculations. This real-life scenario helps in understanding how trigonometric ratios such as sine, cosine, and tangent are related to sides of a right triangle, and how identities connect these ratios.

Some basic formulas:

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}, \quad \cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}, \quad \tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\sin^2\theta + \cos^2\theta = 1, \quad 1 + \tan^2\theta = \sec^2\theta, \quad 1 + \cot^2\theta = \csc^2\theta$$

MCQ Questions

1. If the height of the ramp is 3 m and the base is 4 m, then the hypotenuse of the ramp is:
   (a) 4 m (b) 5 m (c) 6 m (d) 7 m

   Answer: (b) 5 m

   Solution: By Pythagoras theorem, hypotenuse $= \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5$.

2. The sine of the angle of inclination of the ramp is:
   (a) $\frac{3}{4}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{5}{4}$

   Answer: (b) $\frac{3}{5}$

   Solution: $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{3}{5}$.

3. The cosine of the angle of inclination is:
   (a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{5}{4}$ (d) $\frac{5}{3}$

   Answer: (b) $\frac{4}{5}$

   Solution: $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{4}{5}$.

4. Which trigonometric identity is verified using the above values of sine and cosine?
   (a) $\sin^2\theta + \cos^2\theta = 1$ (b) $\sin\theta\cos\theta = \frac{1}{2}$ (c) $\tan^2\theta + 1 = \sec^2\theta$ (d) $\sin^2\theta - \cos^2\theta = 1$

   Answer: (a) $\sin^2\theta + \cos^2\theta = 1$

   Solution: $\sin^2\theta + \cos^2\theta = \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{9}{25} + \frac{16}{25} = \frac{25}{25} = 1$.

5. If $\tan\theta = \frac{3}{4}$ for the ramp's inclination, then $\sec^2\theta$ equals:
   (a) $\frac{16}{9}$ (b) $\frac{25}{9}$ (c) $\frac{25}{16}$ (d) $\frac{9}{25}$

   Answer: (c) $\frac{25}{16}$

   Solution: By identity: $1 + \tan^2\theta = \sec^2\theta$. $1 + \left(\frac{3}{4}\right)^2 = 1 + \frac{9}{16} = \frac{25}{16}$.