

Case Study 2: Ladder Against a Wall and Trigonometric Ratios

Ramesh is a painter who uses a ladder to paint the exterior of a building. The ladder is placed against a wall such that it makes an angle with the ground. The foot of the ladder is at a certain distance from the wall, and the top touches the wall at a certain height. Ramesh wants to make sure that the ladder is safe, so he calculates the angle it makes with the ground using trigonometric ratios. He also checks whether the values satisfy fundamental trigonometric identities. This situation of a ladder forming a right-angled triangle with the wall and ground demonstrates the practical utility of trigonometric ratios and identities in real life. It involves calculating sine, cosine, and tangent of the angle, and verifying their relationships.

Formulas used:

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}, \quad \cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\sin^2 \theta + \cos^2 \theta = 1, \quad 1 + \tan^2 \theta = \sec^2 \theta$$

MCQ Questions

1. A ladder 10 m long is placed so that the foot is 6 m away from the wall. The height at which it touches the wall is:
(a) 7 m (b) 8 m (c) 9 m (d) 10 m

Answer: (b) 8 m

Solution: By Pythagoras theorem, height = $\sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8$.

2. The sine of the angle of elevation of the ladder with the ground is:
(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{5}{4}$

Answer: (a) $\frac{4}{5}$

Solution: $\sin \theta = \frac{8}{10} = \frac{4}{5}$.

3. The cosine of the same angle is:
(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{5}{3}$ (d) $\frac{5}{4}$

Answer: (a) $\frac{3}{5}$

Solution: $\cos \theta = \frac{6}{10} = \frac{3}{5}$.

4. Which identity is satisfied by these values of sine and cosine?
(a) $\sin^2 \theta + \cos^2 \theta = 1$ (b) $\sin \theta \cos \theta = \frac{1}{2}$ (c) $1 + \tan^2 \theta = \csc^2 \theta$ (d) $\sin^2 \theta - \cos^2 \theta = 1$

Answer: (a) $\sin^2 \theta + \cos^2 \theta = 1$

Solution: $\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = \frac{16}{25} + \frac{9}{25} = \frac{25}{25} = 1$.

5. If $\tan \theta = \frac{4}{3}$, then $\sec^2 \theta$ equals:
(a) $\frac{16}{9}$ (b) $\frac{25}{9}$ (c) $\frac{25}{16}$ (d) $\frac{9}{25}$

Answer: (b) $\frac{25}{9}$

Solution: $1 + \tan^2 \theta = \sec^2 \theta$ $1 + \left(\frac{4}{3}\right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$.