

Case Study 2

Ravi, a civil engineer, is planning the layout for a new residential colony. He has identified three key locations for a community center, a park, and a library, which he represents as points on a coordinate plane. The community center is at $A(1, 5)$, the park is at $B(7, 3)$, and the library is at $C(3, -1)$. The local governing body has several questions about the layout and distances between these key locations. They want to ensure that the main walking paths connecting these points are of reasonable length and that the proposed layout covers a significant area. A water supply line needs to be laid from the community center to a point on the path between the park and the library, dividing it in the ratio 3 : 2. Ravi needs to calculate the precise coordinates of this junction point. Additionally, he must determine the area of the triangular plot formed by these three locations to estimate the cost of the landscaping and development. The project manager also asks him to find if the points are collinear, as this would change the entire layout plan.

MCQ Questions

1. What is the distance between the community center (A) and the park (B)? (A) $\sqrt{40}$ units (B) $\sqrt{45}$ units (C) $\sqrt{50}$ units (D) $\sqrt{55}$ units

Answer: (A)

Solution:

$$d = \sqrt{(7-1)^2 + (3-5)^2} = \sqrt{36+4} = \sqrt{40} \text{ units.}$$

2. Find the area of the triangular plot formed by the community center, park, and library. (A) 20 sq. units (B) 22 sq. units (C) 24 sq. units (D) 26 sq. units

Answer: None (Correct value: 16 sq. units)

Solution:

$$\text{Area} = \frac{1}{2} |1(3 - (-1)) + 7(-1 - 5) + 3(5 - 3)| = \frac{1}{2} |4 - 42 + 6| = \frac{1}{2} (-32) = 16 \text{ sq. units.}$$

3. Are the three key locations collinear? (A) Yes, because the area of the triangle is non-zero. (B) No, because the area of the triangle is non-zero. (C) Yes, because the area of the triangle is zero. (D) It cannot be determined from the given data.

Answer: (B)

Solution: Since the area of $\triangle ABC = 16 \neq 0$, the points are not collinear.

4. Find the coordinates of the point that divides the line segment BC in the ratio 3 : 2 internally. (A) $(\frac{27}{5}, \frac{7}{5})$ (B) $(\frac{21}{5}, \frac{3}{5})$ (C) $(\frac{27}{5}, \frac{9}{5})$ (D) $(\frac{21}{5}, \frac{7}{5})$

Answer: None (Correct coordinates: $(\frac{23}{5}, \frac{3}{5})$)

Solution: Using section formula for $B(7, 3)$ and $C(3, -1)$ in ratio 3 : 2:

$$x = \frac{3(3)+2(7)}{5} = \frac{23}{5}, \quad y = \frac{3(-1)+2(3)}{5} = \frac{3}{5}$$

So the coordinates are $(\frac{23}{5}, \frac{3}{5})$.

5. If a new point P is located such that the distance from it to the community center is twice the distance to the library, which of the following is a possible coordinate for P ? (A) $(5, 3)$ (B) $(11, -3)$ (C) $(9, 1)$ (D) $(1, 3)$

Answer: None (Based on calculations, no option fits exactly)

Solution: Condition: $PA = 2PC \implies PA^2 = 4PC^2$ with $A(1, 5), C(3, -1)$. Equation:

$$(x - 1)^2 + (y - 5)^2 = 4[(x - 3)^2 + (y + 1)^2]$$

Simplifies to:

$$3x^2 - 22x + 3y^2 + 18y + 14 = 0$$

Substituting given options shows none satisfies exactly.

www.udgamwelfarefoundation.com