Case Study 1: Understanding Revenue and Profit using Quadratic Equations

Rohit runs a small business that manufactures and sells handcrafted candles. He notices that the profit he earns depends on the number of candles he produces each day. After collecting data for a month, he figures out that the profit P (in rupees) he earns by selling x candles can be modeled by the quadratic equation:

$$P(x) = -2x^2 + 100x - 800$$

This equation helps him understand how to optimize production. The quadratic nature of this equation indicates that there is a maximum profit he can achieve, which corresponds to the vertex of the parabola. He also realizes that if he produces either too few or too many candles, the business will run at a loss. With this model, he wants to understand the most profitable number of candles to produce, break-even points, and interpret real-life decisions based on this mathematical model.

Important Formulas and Concepts

- **Standard form of a quadratic equation:** $ax^2 + bx + c = 0$
- **Vertex of a parabola:** $x = \frac{-b}{2a}$
- **Discriminant:** $D = b^2 - 4ac$
- **Roots of the quadratic equation:** $x = \frac{-b \pm \sqrt{D}}{2a}$

MCQ Questions:

1. What is the number of candles Rohit should produce daily to earn maximum profit?

   (a) 10
   (b) 15
   (c) 25
   (d) 30

   Answer: (c) 25
   Solution: For maximum profit, use vertex formula:

$$x = \frac{-b}{2a} = \frac{-100}{2 \times (-2)} = \frac{-100}{-4} = 25$$

2. What is the maximum profit Rohit can earn?

   (a) Rs.450
   (b) Rs.600
   (c) Rs.425
   (d) Rs.500

   Answer: (b) Rs.600
   Solution: Substitute $x = 25$ in the equation:

   $P(25) = -2(25)^2 + 100(25) - 800 = -1250 + 2500 - 800 = 450$Correction: The answer should be (a) Rs.450

3. What are the break-even points where profit is zero?

   (a) 10, 40
   (b) 20, 30
   (c) 5, 80
   (d) 25, 45

   Answer: (a) 10, 40
   Solution: Solve:

   $-2x^2+100x-800 = 0 \Rightarrow x^2-50x+400 = 0x = \frac{50 \pm \sqrt{(-50)^2 - 4 \cdot 1 \cdot 400}}{2} = \frac{50 \pm \sqrt{2500 - 1600}}{2} = \frac{50 \pm}{2}$

4. What is the nature of roots of the equation $-2x^2 + 100x - 800 = 0$?

   (a) Real and equal
   (b) Real and distinct
   (c) Imaginary
   (d) Cannot be determined

   Answer: (b) Real and distinct
   Solution: Discriminant $D = 100^2 - 4 \cdot (-2) \cdot (-800) = 10000 - 6400 = 3600 > 0$. Hence, roots are real and distinct.

5. Which of the following quadratic equations has equal roots?

   (a) $x^2 + 4x + 5 = 0$
   (b) $x^2 + 6x + 9 = 0$
   (c) $x^2 + 2x + 3 = 0$
   (d) $x^2 - 7x + 10 = 0$

   Answer: (b) $x^2 + 6x + 9 = 0$
   Solution:

   For equal roots, the discriminant $D = b^2 - 4ac = 0$

   (a) $D = 4^2-4(1)(5) = 16-20 = -4$ (not equal roots)(b) $D = 6^2-4(1)(9) = 36-36 = 0$ **equal roots**(c) D

   Only option (b) has D = 0, hence it has equal roots.