Case Study 2

A school is planning to build a rectangular flower garden. The principal wants the garden's length to be 4 metres more than its width. The area of the garden is expected to be 96 square metres. To find the appropriate dimensions, the students decide to model the situation using a quadratic equation. They learn that converting the geometric condition into an algebraic equation leads to a quadratic expression that can be solved either by factorization or by completing the square. Using both methods allows them to verify their answer. This real-life context helps them understand how quadratic equations can represent physical problems, such as dimensions and area.

Key Concepts and Formulas

- Let width be x, then length $= x + 4$
- Area of rectangle = length $\times$ width
- Standard form: $ax^2 + bx + c = 0$
- Factorization and Completing the Square methods:

$$x^2 + 4x - 96 = 0$$

- Completing the square: Convert to a perfect square trinomial.

MCQ Questions:

1. What quadratic equation represents the area of the rectangular garden?

   (a) $x^2 + 4x + 96 = 0$
   (b) $x^2 - 4x - 96 = 0$
   (c) $x^2 + 4x - 96 = 0$
   (d) $x^2 - 4x + 96 = 0$

   Answer: (c) $x^2 + 4x - 96 = 0$
   Solution:

   $$\text{Let width be } x, \text{ length is } x + 4, \text{ area is } 96 \Rightarrow x(x + 4) = 96 \Rightarrow x^2 + 4x - 96 = 0$$

2. Solve the equation using factorization. What is the width of the garden?

   (a) 8 metres
   (b) 12 metres
   (c) 6 metres
   (d) 10 metres

   Answer: (a) 8 metres
   Solution:

   $x^2+4x-96 = 0 \Rightarrow (x+12)(x-8) = 0 \Rightarrow x = -12$ or 8Negative width is not possible, so width = 8 m

3. What is the corresponding length of the garden?

   (a) 10 metres

(b) 12 metres

(c) 8 metres

(d) 14 metres

Answer: (b) 12 metres
Solution:

$$\text{Length} = x + 4 = 8 + 4 = 12 \text{ m}$$

4. Solve the equation $x^2 + 4x - 96 = 0$ by completing the square. What value of x do you get?

(a) 6

(b) 8

(c) 12

(d) 10

Answer: (b) 8
Solution:

$x^2+4x-96 = 0 \Rightarrow x^2+4x = 96 \Rightarrow x^2+4x+4 = 100 \Rightarrow (x+2)^2 = 100 \Rightarrow x+2 = \pm 10 \Rightarrow x = 8$ or -12 Only

5. What is the area of the rectangular garden based on the final dimensions?

(a) 96 square metres

(b) 104 square metres

(c) 112 square metres

(d) 88 square metres

Answer: (a) 96 square metres
Solution:

$$\text{Width } = 8, \text{ Length } = 12, \text{ Area } = 8 \times 12 = 96 \text{ sq m}$$