Case Study 1

Rohit, a Class 10 student from a CBSE school, is exploring polynomials during his summer project. He is particularly fascinated by how polynomials behave graphically and algebraically. As a part of his research, he plotted several quadratic polynomials using a graphing software and observed that the points where the graphs intersect the x-axis are known as the zeros of the polynomials. He also noted that a polynomial of degree n can have at most n real zeros. Curious to test this idea further, he started analyzing the number and nature of zeros for different quadratic polynomials, including cases where no real zeros exist. He began documenting how the coefficients influence the number of real zeros and their signs. Help Rohit answer the following questions:

1. How many zeros does the polynomial $f(x) = x^2 - 5x + 6$ have?

   (a) 1

   (b) 2

   (c) 0

   (d) 3

   Answer: (b)

   Solution: The given quadratic polynomial can be factored as:

   $$f(x) = x^2 - 5x + 6 = (x - 2)(x - 3)$$

   So, the zeros are $x = 2$ and $x = 3$. Hence, the number of zeros is 2.

2. Which of the following polynomials has only one zero?

   (a) $x^2 + 2x + 1$

   (b) $x^2 - 2x + 2$

   (c) $x^2 - 4$

(d) $x^2 - 3x + 2$

Answer: (a)

Solution: $x^2 + 2x + 1 = (x + 1)^2$ has a repeated zero at $x = -1$, so it has only one real zero with multiplicity 2.

3. What are the zeros of the polynomial $f(x) = x^2 + 4x + 3$?

(a) $-1, -3$

(b) $1, 3$

(c) $-3, -1$

(d) $3, 1$

Answer: (c)

Solution:

$$f(x) = x^2 + 4x + 3 = (x + 1)(x + 3)$$

So, the zeros are -1 and -3.

4. Which of the following polynomials has no real zeros?

(a) $x^2 - 4x + 4$

(b) $x^2 + 6$

(c) $x^2 - 1$

(d) $x^2 + 3x + 2$

Answer: (b)

Solution: $x^2 + 6$ has no real roots because the discriminant $D = 0^2 - 4 \cdot 1 \cdot 6 = -24$ is negative.

5. Which condition must be true for a quadratic polynomial $ax^2 + bx + c$ to have exactly one real zero?

(a) $b^2 - 4ac > 0$

(b) $b^2 - 4ac < 0$

(c) $b^2 - 4ac = 0$

(d) $a = 0$

Answer: (c)

Solution: A quadratic polynomial has exactly one real zero when the discriminant $D = b^2 - 4ac = 0$, which results in a repeated root.