Case Study 3

A group of engineers is working on the design of a water fountain in a newly constructed park. The stream of water from the fountain follows a parabolic path, and the engineers model this using quadratic polynomials. Suppose the water jet touches the ground at two different points, creating a curve in the form of a parabola. The equation of this parabola can be represented by $p(x) = ax^2 + bx + c$, where the zeros of the polynomial represent the points at which the stream touches the ground. The maximum height of the fountain is determined by analyzing the vertex of the parabola, and the symmetry of the stream depends on the relationship between the zeros of the polynomial. By studying these mathematical models, the engineers can adjust the design to ensure the water stream looks aesthetic and lands precisely within the fountain boundary. For quadratic polynomials, the important relations are:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a},$$

where α and β are the zeros of the polynomial. Let us answer the following questions based on this situation.

MCQ Questions

1. The polynomial representing the fountain's water stream is $p(x) = x^2 - 6x + 8$. The sum of the zeros is:

   (a) 2

   (b) 4

   (c) 6

   (d) 8

   Answer: (c) 6

   Solution: For $p(x) = x^2 - 6x + 8$, $\alpha + \beta = -\frac{-6}{1} = 6$.

2. If the zeros of $p(x) = 2x^2 - 5x + 2$ are α and β, then the product $\alpha\beta$ is:

   (a) $\frac{5}{2}$
   (b) 2
   (c) $\frac{2}{2}$
   (d) 1

   Answer: (b) 2
   Solution: $\alpha\beta = \frac{c}{a} = \frac{2}{2} = 1$.
   Correct answer is (d) 1, not (b).

3. The polynomial having zeros 2 and 5 is:

   (a) $x^2 + 7x + 10$
   (b) $x^2 - 7x + 10$
   (c) $x^2 - 10x + 7$
   (d) $x^2 + 10x + 7$

   Answer: (b) $x^2 - 7x + 10$
   Solution: $(x - 2)(x - 5) = x^2 - 7x + 10$.

4. For $p(x) = x^2 - 3x - 10$, if α and β are the zeros, then $\alpha^2 + \beta^2 =$:

   (a) 9
   (b) 19
   (c) 13
   (d) 29

   Answer: (b) 19
   Solution: $\alpha + \beta = 3$, $\alpha\beta = -10$. Then $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 9 - 2(-10) = 29$. Correct answer is (d) 29.

5. The quadratic polynomial whose zeros are $\frac{1}{4}$ and $\frac{1}{2}$ is:

   (a) $4x^2 - 3x + \frac{1}{2}$

(b) $8x^2 - 6x + 1$

(c) $4x^2 + 3x + 1$

(d) $2x^2 - 3x + 1$

Answer: (b) $8x^2 - 6x + 1$

Solution: Required polynomial $= \left(x - \frac{1}{4}\right)\left(x - \frac{1}{2}\right) = x^2 - \frac{3}{4}x + \frac{1}{8}$. Multiplying through by 8 gives $8x^2 - 6x + 1$.