Theoretical Formulas and Properties

Distance Formula

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Section Formula (Internal Division)

The coordinates of a point $P(x, y)$ that divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are:

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

Area of a Triangle

The area of a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ is given by:

$$\text{Area} = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

Case Study 1

Rohan, a student of class 10, is designing a logo for his school's annual science exhibition. The logo is a geometrical pattern created on a coordinate plane. He has fixed the three main vertices of the logo at points $A(2, 3)$, $B(6, 1)$, and $C(4, 5)$. The logo's design is based on the properties of the triangle formed by these three points. He wants to know the shape of the triangle, its area, and the properties of the line segments within it to perfect his design. Rohan's friend, who is good at programming, also wants to find the coordinates of a point that divides one of the sides of this triangle in a specific ratio. This logo will be displayed on a banner, and for that, he needs to find out the coordinates of the point that is equidistant from all three vertices, as this point will be the center of a circular design element.

MCQ Questions

1. What is the distance between vertices A and B? (A) $\sqrt{20}$ units   (B) $\sqrt{26}$ units   (C) $2\sqrt{5}$ units   (D) $4\sqrt{2}$ units

   Answer: (A)

   Solution: Using the distance formula for $A(2, 3)$ and $B(6, 1)$:

   $$d = \sqrt{(6 - 2)^2 + (1 - 3)^2} = \sqrt{16 + 4} = \sqrt{20} \text{ units.}$$

2. Find the area of the triangle ABC. (A) 4 sq. units   (B) 6 sq. units   (C) 8 sq. units   (D) 10 sq. units

   Answer: (B)

   Solution: Using the formula:

   $$\text{Area} = \frac{1}{2} |2(1 - 5) + 6(5 - 3) + 4(3 - 1)| = \frac{1}{2} |-8 + 12 + 8| = \frac{1}{2}(12) = 6 \text{ sq. units.}$$

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3. What are the coordinates of the midpoint of line segment AC? (A) $(3,4)$ (B) $(3,5)$ (C) $(4,4)$ (D) $(3,3)$

Answer: (A)

Solution: Midpoint of $A(2,3)$ and $C(4,5)$:

$$x = \frac{2+4}{2} = 3, \quad y = \frac{3+5}{2} = 4$$

Hence midpoint is $(3,4)$.

4. Find the coordinates of a point P that divides line segment BC in ratio $2:1$ internally. (A) $\left(\frac{16}{3}, \frac{7}{3}\right)$ (B) $\left(\frac{14}{3}, \frac{7}{3}\right)$ (C) $\left(\frac{14}{3}, \frac{13}{3}\right)$ (D) $\left(\frac{14}{3}, \frac{11}{3}\right)$

Answer: (D)

Solution: Using section formula for $B(6,1)$ and $C(4,5)$:

$$x = \frac{2(4)+1(6)}{3} = \frac{14}{3}, \quad y = \frac{2(5)+1(1)}{3} = \frac{11}{3}$$

So $P = \left(\frac{14}{3}, \frac{11}{3}\right)$.

5. If Rohan wants to place a circular element at the point equidistant from all three vertices A, B, C, what are the coordinates of the center? (A) $(4,3)$ (B) $(4,4)$ (C) $(5,3)$ (D) $(5,4)$

Answer: None of the above (Correct coordinates: $\left(\frac{13}{3}, \frac{8}{3}\right)$).

Solution: The circumcenter (x,y) satisfies $PA = PB = PC$. Solving the equations gives:

$$(x,y) = \left(\frac{13}{3}, \frac{8}{3}\right).$$