Case Study 3

Two students, Ankit and Meera, were participating in a physics experiment involving the motion of an object thrown vertically upwards. They derived the equation to calculate the time t (in seconds) taken by the object to reach the ground after being thrown from a height of 80 metres with an initial velocity of 20 metres per second. The equation they derived was:

$$5t^2 - 20t + 80 = 0$$

They were asked to interpret the nature of the time values the object could have. Their physics teacher explained that the discriminant of the quadratic equation plays a critical role in determining the nature of the roots and whether the physical scenario is possible in real life. They realized that the concept of discriminant and the nature of roots is not only a part of algebra but also a vital part of understanding real-world physical phenomena.

Important Concepts and Formulas

- General quadratic equation: $ax^2 + bx + c = 0$
- Discriminant: $D = b^2 - 4ac$
- Nature of roots:
  - $D > 0$: Real and distinct roots
  - $D = 0$: Real and equal roots
  - $D < 0$: No real roots (imaginary roots)

MCQ Questions:

1. What is the value of the discriminant for the equation $5t^2 - 20t + 80 = 0$?

   (a) 400
   (b) -400
   (c) 0
   (d) 100

   Answer: (b) -400
   Solution:

   $D = (-20)^2 - 4 \cdot 5 \cdot 80 = 400 - 1600 = -1200 \Rightarrow$ Corrected option: Answer is not listed properly. Correct a

2. What does the discriminant tell us about the nature of roots?

   (a) Real and equal
   (b) Real and distinct
   (c) Imaginary
   (d) Cannot be determined

   Answer: (c) Imaginary
   Solution: Since $D < 0$, the roots are not real (they are imaginary).

3. What does the result imply in the context of the physical motion?

(a) The object hits the ground at two times

(b) The object never hits the ground

(c) The object is in free-fall motion

(d) The velocity was zero

Answer: (b) The object never hits the ground
Solution: Imaginary roots indicate no real solution for time, so the object doesn't reach the ground as per this equation—likely due to incorrect modelling or a constraint in initial conditions.

4. If the equation were $5t^2 - 20t + 15 = 0$, what would be the nature of roots?

   (a) Real and distinct
   (b) Real and equal

   (c) Imaginary
   (d) None of these

   Answer: (a) Real and distinct
   Solution:

$$D = (-20)^2 - 4 \cdot 5 \cdot 15 = 400 - 300 = 100 > 0 \Rightarrow \text{Real and distinct roots}$$

5. For which condition will the roots of $ax^2 + bx + c = 0$ be equal?

   (a) $b^2 = 4ac$
   (b) $b^2 > 4ac$

   (c) $b^2 < 4ac$
   (d) $a = 0$

   Answer: (a) $b^2 = 4ac$
   Solution: This condition gives $D = 0$, which results in real and equal roots.