Case Study 2: Relationship Between Zeros and Coefficients

Ankita, a high school student preparing for her board exams, is revising quadratic polynomials. Her teacher explains that if α and β are the zeros of a quadratic polynomial $ax^2 + bx + c$, then these zeros have a specific relationship with the coefficients. She learns the following key formulas:

- Sum of the zeros $\alpha + \beta = -\frac{b}{a}$
- Product of the zeros $\alpha \cdot \beta = \frac{c}{a}$

Using these relationships, Ankita is able to form a quadratic polynomial when the zeros are known, using the formula:

$$f(x) = x^2 - (\text{sum of zeros})x + (\text{product of zeros})$$

With this knowledge, she works on the following problems:

1. If the zeros of the polynomial $x^2 - 7x + 10$ are α and β, what is $\alpha + \beta$ and $\alpha \cdot \beta$?

   (a) $7, 10$

   (b) $-7, 10$

   (c) $7, -10$

   (d) $-7, -10$

   Answer: (a)

   Solution:

   $$\alpha + \beta = -\frac{-7}{1} = 7, \quad \alpha \cdot \beta = \frac{10}{1} = 10$$

   Hence, the correct answer is option (a).

2. Find the quadratic polynomial whose zeros are 3 and 4.

(a) $x^2 - 7x + 12$

(b) $x^2 - 12x + 7$

(c) $x^2 + 7x + 12$

(d) $x^2 + 12x - 7$

Answer: (a)

Solution: Sum $= 3 + 4 = 7$, Product $= 3 \cdot 4 = 12$

So the polynomial is:

$$x^2 - 7x + 12$$

Option (a) matches correctly.

3. If the sum and product of the zeros of a quadratic polynomial are 5 and 6 respectively, what is the polynomial?

(a) $x^2 - 5x + 6$

(b) $x^2 + 5x + 6$

(c) $x^2 - 6x + 5$

(d) $x^2 + 6x + 5$

Answer: (a)

Solution:

$$f(x) = x^2 - (\text{sum})x + \text{product} = x^2 - 5x + 6$$

Option (a) is correct.

4. For the polynomial $2x^2 - 4x + 3$, what is the sum and product of its zeros?

(a) $2, 3$

(b) $2, \frac{3}{2}$

(c) $2, \frac{3}{4}$

(d) $\frac{4}{2}, \frac{3}{2}$

Answer: (b)

Solution:

$$\text{Sum of zeros} = -\frac{-4}{2} = 2, \quad \text{Product of zeros} = \frac{3}{2}$$

Option (b) is correct.

5. What is the quadratic polynomial whose sum and product of zeros are both equal to 1?

(a) $x^2 - x + 1$

(b) $x^2 + x + 1$

(c) $x^2 - 1x + 1$

(d) $x^2 - 2x + 1$

Answer: (c)

Solution:

$$f(x) = x^2 - (1)x + 1 = x^2 - x + 1$$

Both options (a) and (c) are equivalent. However, (c) writes the coefficient explicitly as $-1x$, which aligns better with clarity. So, both are correct representations, but we will consider (c) based on form.