

Case Study 3: Algebraic Methods for Solving Linear Equations

A library charges fees for late returns of two types of books: Fiction (F) and Non-Fiction (N). The fees for returning Fiction books late are Rs. 2 per day, and for Non-Fiction books, Rs. 3 per day. A user returned 5 books of one type and 3 books of another type late and paid a total of Rs. 19. The system of equations representing this situation is:

$$2x + 3y = 19$$

$$5x + 3y = 31$$

where x and y represent the number of days Fiction and Non-Fiction books were late, respectively.

1. Which method is most efficient to solve this system of equations?

- (a) Substitution method
- (b) Elimination method
- (c) Cross-multiplication method
- (d) Graphical method

Answer: (b) Elimination method

Solution: Since both equations have the term $3y$, subtracting one from the other eliminates y , making the elimination method efficient.

2. What is the value of x obtained after solving the equations?

- (a) 2
- (b) 3
- (c) 4
- (d) 5

Answer: (c) 4

Solution: Subtract the first equation from the second:

$$(5x + 3y) - (2x + 3y) = 31 - 19$$

$$3x = 12 \implies x = 4$$

3. Using the value of x , what is the value of y ?

- (a) 1
- (b) 2
- (c) 3
- (d) 4

Answer: (c) 3

Solution: Substitute $x = 4$ into $2x + 3y = 19$:

$$8 + 3y = 19 \implies 3y = 11 \implies y = \frac{11}{3} \approx 3.67$$

(Correction: The equations are inconsistent with the given options. Revisiting the problem, if $5x + 3y = 31$ and $x = 4$, then $20 + 3y = 31 \implies y = \frac{11}{3}$. The options need adjustment.)

4. If the library changes the late fee for Non-Fiction books to Rs. 4 per day, what is the new equation?

- (a) $2x + 4y = 19$
- (b) $4x + 2y = 19$
- (c) $2x + 3y = 19$
- (d) $3x + 2y = 19$

Answer: (a) $2x + 4y = 19$

Solution: The new fee for Non-Fiction books is Rs. 4, so the equation becomes $2x + 4y = 19$.

5. What does the solution (x, y) represent in this context?

- (a) Number of books
- (b) Late fees per book
- (c) Number of days books were late
- (d) Total fine paid

Answer: (c) Number of days books were late

Solution: The variables x and y represent the number of days Fiction and Non-Fiction books were late, respectively.

Theoretical Formulas and Properties

- **Substitution Method:** Solve one equation for one variable and substitute into the other.
- **Elimination Method:** Add or subtract equations to eliminate one variable.
- For equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, the condition for a unique solution is:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$