**Case Study 1:**

Shivani is an architecture student who is working on designing a unique garden layout for a school campus. The boundary of the garden on one side follows the parabolic curve given by $y = x^2$, while a decorative fence runs along the line $y = 4$, which acts as a horizontal limit to the garden space. Shivani wants to calculate the exact area enclosed between the curve $y = x^2$ and the line $y = 4$ to help her order the correct amount of grass turf for landscaping. She decides to use the concept of definite integration to determine the area between these two curves. The points of intersection will define the limits of integration. This practical application demonstrates how integration is used in architectural and civil engineering designs to estimate area, resource usage, and cost planning.

**Concepts and Formulae Used:**

- Area between two curves:
$$\text{Area} = \int_a^b [f(x) - g(x)]\,dx$$
where $f(x) \geq g(x)$ on the interval $[a, b]$.
- Find the points of intersection of the two curves to determine the limits of integration.

**MCQ Questions:**

1. What are the points of intersection of the curves $y = x^2$ and $y = 4$?

   (a) $x = -2, x = 2$

   (b) $x = 0, x = 4$

   (c) $x = -4, x = 4$

   (d) $x = -1, x = 1$

   **Answer:** (a)
   **Solution:** Set $x^2 = 4 \Rightarrow x = \pm 2$

2. What is the correct integrand for the area between the line and the parabola?

   (a) $\int_{-2}^{2}(4 + x^2)\,dx$

   (b) $\int_{-2}^{2}(x^2 - 4)\,dx$

   (c) $\int_{-2}^{2}(4 - x^2)\,dx$

   (d) $\int_{-2}^{2}(x^2 + 4)\,dx$

   **Answer:** (c)
   **Solution:** Since $y = 4$ is above $y = x^2$, the integrand is $4 - x^2$

3. What is the value of $\int_{-2}^{2} x^2\,dx$?

   (a) $\dfrac{16}{3}$

(b) $\frac{8}{3}$

(c) 0

(d) 4

**Answer:** (a)
**Solution:** $\int_{-2}^{2} x^2\,dx = 2\int_0^2 x^2\,dx = 2 \cdot \frac{8}{3} = \frac{16}{3}$

4. What is the total area enclosed between the curve and the line?

(a) $\frac{32}{3}$

(b) $\frac{40}{3}$

(c) $\frac{16}{3}$

(d) $\frac{28}{3}$

**Answer:** (a)
**Solution:**

$$\text{Area} = \int_{-2}^{2} (4 - x^2)\,dx = \int_{-2}^{2} 4\,dx - \int_{-2}^{2} x^2\,dx = 8 \cdot 4 - \frac{16}{3} = 16 - \frac{16}{3} = \frac{48 - 16}{3} = \frac{32}{3}$$

5. What property of the integrand can be used to simplify $\int_{-2}^{2}(4 - x^2)\,dx$?

(a) Linearity of integrals

(b) Odd function property

(c) Symmetry about y-axis

(d) Fundamental theorem of calculus

**Answer:** (c)
**Solution:** Both functions $x^2$ and the constant 4 are even functions, so symmetry about the y-axis allows calculation over $[0, 2]$ and doubling it.