Case Study: 4

Two artists, Ravi and Meera, are designing murals on separate walls, both involving the area enclosed between curves. Ravi chooses a region between the line $y = x + 1$ and the parabola $y = x^2$ from $x = -1$ to $x = 2$. Meera chooses a different region but wants to compare the complexity of integration. Both artists decide to calculate the area enclosed between the given curves to estimate the paint required. They use the concept of definite integration to find the difference between the upper and lower curves over the interval of interest.

Concepts and Formulae Used:

- Area between two curves:

$$\text{Area} = \int_a^b [f(x) - g(x)]\,dx, \text{ where } f(x) \geq g(x)$$

- Graphical intersection is useful to identify limits and which function is above.
- The area is always taken as a positive quantity.

MCQ Questions:

1. What is the point of intersection of the curves $y = x^2$ and $y = x + 1$?

   (a) $x = -2, 1$

   (b) $x = -1, 2$

   (c) $x = 0, 1$

   (d) $x = -1, 1$

   Answer: (b)
   Solution: Set $x^2 = x + 1 \Rightarrow x^2 - x - 1 = 0 \Rightarrow x = -1, 2$

2. Which function lies above in the interval $[-1, 2]$?

   (a) $y = x^2$

   (b) $y = x + 1$

   (c) Both are equal

   (d) Cannot be determined

   Answer: (b)
   Solution: For x in $[-1, 2]$, $x + 1 > x^2$ for most values (verify with sample points)

3. What is the integral expression to compute the required area?

   (a) $\int_{-1}^{2}(x + 1 - x^2)\,dx$

   (b) $\int_{-1}^{2}(x^2 - x - 1)\,dx$

(c) $\int_{-1}^{2} |x^2 - (x+1)|\, dx$

(d) $\int_{-1}^{2} (x^2 + x + 1)\, dx$

Answer: (a)
Solution: Upper curve is $x+1$, lower is x^2, so integral is $\int_{-1}^{2}(x+1-x^2)dx$

4. Evaluate $\int_{-1}^{2}(x+1-x^2)\, dx$.

   (a) $\frac{9}{2}$

   (b) $\frac{11}{6}$

   (c) $\frac{9}{4}$

   (d) $\frac{10}{3}$

   Answer: (b)
   Solution:

$$\int_{-1}^{2}(x+1-x^2)\, dx = \left[\frac{x^2}{2} + x - \frac{x^3}{3}\right]_{-1}^{2} = \frac{11}{6}$$

5. Why is it important to check which function is greater over the interval?

   (a) To avoid division

   (b) To apply symmetry

   (c) To ensure positive area

   (d) To reduce calculation steps

   Answer: (c)
   Solution: Area is a scalar quantity. Using the wrong order may result in negative values.