Case Study: 5

In a school mathematics competition, students were given the task of calculating the area bounded by the curve $y = \sin x$ and the x-axis from $x = -\pi$ to $x = \pi$. Anika and Sohail approached the problem differently. Anika used the property of symmetry of the sine function, while Sohail directly computed the definite integral. The teacher encouraged both approaches, pointing out that symmetric functions can simplify integration, especially when dealing with even or odd functions.

Concepts and Formulae Used:

- If $f(x)$ is odd: $\int_{-a}^{a} f(x)dx = 0$
- If $f(x)$ is even: $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$
- $\sin x$ is an odd function: $\sin(-x) = -\sin(x)$
- Area under curve $y = f(x)$ and x-axis: $\int_{a}^{b} |f(x)|dx$

MCQ Questions:

1. Which of the following describes the symmetry of $y = \sin x$?

   (a) Even function

   (b) Odd function

   (c) Neither

   (d) Periodic but not symmetric

   Answer: (b)
   Solution: Since $\sin(-x) = -\sin(x)$, it is an odd function.

2. What is the value of $\int_{-\pi}^{\pi} \sin x \, dx$?

   (a) 0

   (b) 2

   (c) -2

   (d) π

   Answer: (a)
   Solution: Since $\sin x$ is odd, the integral over symmetric interval is 0.

3. What is the total area bounded by $y = \sin x$ and the x-axis from $x = -\pi$ to $x = \pi$?

   (a) 2

   (b) 0

   (c) $\int_{0}^{\pi} \sin x \, dx$

(d) $2\int_0^{\pi} \sin x\, dx$

Answer: (d)
Solution: Use absolute value: total area $= 2\int_0^{\pi} \sin x\, dx$

4. Calculate $\int_0^{\pi} \sin x\, dx$.

(a) 1

(b) 2

(c) π

(d) 0

Answer: (b)
Solution: $\int_0^{\pi} \sin x\, dx = [-\cos x]_0^{\pi} = -(-1) + 1 = 2$

5. What is the total area under $y = \sin x$ from $x = -\pi$ to $x = \pi$?

(a) 1

(b) 2

(c) 4

(d) π

Answer: (c)
Solution: Area is $2\int_0^{\pi} \sin x\, dx = 2 \times 2 = 4$