Case Study 1: Understanding Vectors and Their Types

In a physics laboratory, a team of students is studying the motion of an object on a smooth surface. They record the displacement of the object using directed line segments and understand that these can be represented as vectors. The team observes how vectors behave when applied in different directions and magnitudes. They begin by identifying the basic concepts — such as zero vector, unit vector, and position vector. During their observations, they also discuss the concept of equal and opposite vectors and how two vectors can be collinear or coplanar. They use arrows on graph paper to draw and represent these vectors and verify the properties of vector addition and scalar multiplication.

Theory and Formulae Related to Vectors:

- A vector is a quantity that has both magnitude and direction.
- A zero vector has zero magnitude: $\vec{0} = 0\hat{i} + 0\hat{j} + 0\hat{k}$.
- A unit vector has magnitude 1: $|\vec{a}| = 1$.
- Two vectors are equal if they have the same magnitude and direction.
- Collinear vectors: $\vec{a} = \lambda\vec{b}$ for some scalar λ.
- Position vector of a point $P(x, y, z)$: $\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$.

MCQ Questions

1. Which of the following is a unit vector?

   (a) $\hat{i} + \hat{j}$

   (b) $\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$

   (c) $\hat{i} + 2\hat{j}$

   (d) $2\hat{i} - 2\hat{j}$

   Answer: (b)
   Solution: A unit vector must have magnitude 1. Check for (b):
   $\left|\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})\right| = \frac{1}{\sqrt{2}}\sqrt{1^2 + 1^2} = \frac{1}{\sqrt{2}} \cdot \sqrt{2} = 1$

2. If $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, what is the magnitude of $\vec{a}$?

   (a) 9

   (b) $\sqrt{29}$

   (c) $\sqrt{13}$

   (d) 5

Answer: (b)
Solution: $|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$

3. Which of the following is a zero vector?

   (a) $\vec{a} = \hat{i} + \hat{j}$

   (b) $\vec{a} = 0\hat{i} + 0\hat{j} + 0\hat{k}$

   (c) $\vec{a} = -\hat{i} - \hat{j}$

   (d) $\vec{a} = \hat{k}$

   Answer: (b)
   Solution: Zero vector has zero magnitude: $|\vec{a}| = \sqrt{0^2 + 0^2 + 0^2} = 0$

4. If $\vec{a} = \hat{i} + 2\hat{j}$ and $\vec{b} = 2\hat{i} + 4\hat{j}$, then which of the following is true?

   (a) $\vec{a}$ and $\vec{b}$ are perpendicular

   (b) $\vec{a}$ and $\vec{b}$ are unit vectors

   (c) $\vec{a}$ and $\vec{b}$ are collinear

   (d) None of these

   Answer: (c)
   Solution: $\vec{b} = 2\vec{a}$, so $\vec{a}$ and $\vec{b}$ are scalar multiples $\Rightarrow$ collinear

5. Which statement is correct for equal vectors?

   (a) They have the same magnitude but different directions

   (b) They have the same direction but different magnitudes

   (c) They have same magnitude and direction

   (d) They are always unit vectors

   Answer: (c)
   Solution: Equal vectors must have same direction and magnitude by definition