Case Study 3: Online Shopping and Customer Satisfaction (Probability)

Case Study Description:

A large e-commerce platform monitors customer activity and satisfaction. They know that 70% of all customers who visit their website place an order. Of the customers who place an order, 90% express high satisfaction with the delivery and product quality. However, of the customers who **do not** place an order, only 20% express high satisfaction (perhaps due to finding what they needed elsewhere or enjoying the browsing experience). The platform is interested in understanding the overall satisfaction rate and the relationship between placing an order and expressing high satisfaction. Furthermore, they decide to track a group of 5 customers who visit the site and analyze the probability distribution of those who express high satisfaction. This scenario is crucial for applying concepts like **conditional probability**, checking for **independent events**, and calculating the **mean** and **variance of a random variable** within a practical business context. The company must determine if satisfaction is significantly influenced by whether a purchase was made. If the events are not independent, placing an order is a strong predictor of satisfaction.

Let O be the event that a customer places an order, and S be the event that a customer expresses high satisfaction. We are given the following probabilities: $P(O) = 0.70$, $P(S|O) = 0.90$, and $P(S|O') = 0.20$. We need to use the Multiplication Theorem and the Total Probability Theorem to find the probability of overall satisfaction and then check if the events O and S are independent. For the random variable analysis, we will assume the 5 customers act independently.

MCQ Questions (5 Questions)

1. What is the probability that a customer **does not** place an order and also expresses high satisfaction?

   (a) 0.20
   (b) 0.06
   (c) 0.63
   (d) 0.14

   Answer: (b)

   Solution: Let O' be the event the customer does not place an order. We need to find $P(O' \cap S)$. We use the **Multiplication Theorem of Probability**:

   $$P(O' \cap S) = P(O') \cdot P(S|O')$$

   We have $P(O) = 0.70$, so $P(O') = 1 - 0.70 = 0.30$. We are given $P(S|O') = 0.20$.

   $$P(O' \cap S) = 0.30 \times 0.20 = 0.06$$

2. What is the overall probability that a randomly selected customer expresses high satisfaction ($P(S)$)? (Total Probability)

   (a) 0.69
   (b) 0.85
   (c) 0.75
   (d) 0.90

Answer: (a)

Solution: The total probability of high satisfaction, $P(S)$, is given by the **Theorem of Total Probability**:

$$P(S) = P(O)P(S|O) + P(O')P(S|O')$$

$P(O)P(S|O) = 0.70 \times 0.90 = 0.63$ (Satisfaction with order) $P(O')P(S|O') = 0.30 \times 0.20 = 0.06$ (Satisfaction without order)

$$P(S) = 0.63 + 0.06 = 0.69$$

3. Are the events O (Placing an order) and S (High satisfaction) independent?

   (a) Yes, because $P(S|O) \neq P(S)$.
   (b) Yes, because $P(O \cap S) = P(O)P(S)$.
   (c) No, because $P(S|O) \neq P(S)$.
   (d) No, because $P(S|O) = P(S)$.

   Answer: (c)

   Solution: Two events O and S are **independent** if $P(S|O) = P(S)$. We have $P(S|O) = 0.90$ (given). From Q2, we have $P(S) = 0.69$. Since $0.90 \neq 0.69$, the events O and S are **not independent**. Placing an order significantly increases the probability of high satisfaction. The correct condition for non-independence is $P(S|O) \neq P(S)$.

4. A customer expresses high satisfaction. What is the probability that they **did not** place an order? (Bayes' Theorem)

   (a) $\frac{6}{69}$
   (b) $\frac{63}{69}$
   (c) $\frac{2}{30}$
   (d) $\frac{7}{10}$

   Answer: (a)

   Solution: We need to find the conditional probability $P(O'|S)$, which is calculated using **Bayes' Theorem**:

   $$P(O'|S) = \frac{P(O' \cap S)}{P(S)}$$

   From Q1, $P(O' \cap S) = 0.06$. From Q2, $P(S) = 0.69$.

   $$P(O'|S) = \frac{0.06}{0.69} = \frac{6}{69} = \frac{2}{23}$$

   The option $\frac{6}{69}$ is present and is mathematically equivalent to $\frac{2}{23}$.

5. For the group of 5 customers (independent trials), let X be the random variable representing the number of customers who express high satisfaction. What is the **variance** of X? (Random Variable and Variance)

   (a) $5 \times 0.69 \times 0.31$
   (b) $5 \times 0.70 \times 0.30$
   (c) $5 \times (0.69)^2$

(d) 5×0.69

Answer: (a)

Solution: The random variable X follows a Binomial distribution $B(n, p)$ since we have $n = 5$ independent trials (customers) and the probability of success is $p = P(S)$. From Q2, $p = P(S) = 0.69$. The probability of failure is $q = 1 - p = 1 - 0.69 = 0.31$. The **Variance** of a Binomial distribution is given by $\text{Var}(X) = n \cdot p \cdot q$.

$$\text{Var}(X) = 5 \times 0.69 \times 0.31$$