Case Study 1: Production Reliability (Probability)

Case Study Description:

A company produces **LED bulbs** in two different plants, **Plant I** and **Plant II**. Plant I manufactures 60% of the total bulbs, and Plant II manufactures the remaining 40%. The quality control department has determined the probability of a bulb being **defective** based on the plant of origin. The probability that a bulb produced by Plant I is defective is 5%, and the probability that a bulb produced by Plant II is defective is 2%. The company packages all the bulbs together for sale in the market. A buyer, Ramesh, purchases a large box of these LED bulbs. He is concerned about the reliability and quality of the production process. He wants to use the principles of probability to analyze the production data.

Ramesh decides to randomly select one bulb from the box to test its quality. He is interested in finding the probability that the selected bulb is defective. Furthermore, if he finds a selected bulb to be defective, he wants to determine the probability that it came from a specific plant, say Plant I. This is a classic example where the concepts of **conditional probability**, **multiplication theorem of probability**, and **Bayes' Theorem** are essential for understanding the overall quality and the source of potential defects. The relative contribution of each plant to the total production and their individual defect rates influence the final probability of finding a defective item.

Let E_1 be the event that the bulb is produced by **Plant I**, and E_2 be the event that the bulb is produced by **Plant II**. Let A be the event that the selected bulb is **defective**. We are given the following probabilities: $P(E_1) = 0.60$, $P(E_2) = 0.40$, $P(A|E_1) = 0.05$, and $P(A|E_2) = 0.02$. Ramesh needs to calculate the total probability $P(A)$ and the posterior probability $P(E_1|A)$.

MCQ Questions (5 Questions)

1. What is the probability that a randomly selected bulb from the box is produced by Plant II and is defective?

   (a) 0.020
   (b) 0.008
   (c) 0.030
   (d) 0.400

   Answer: (b)

   Solution: Let E_2 be the event that the bulb is from Plant II, and A be the event that the bulb is defective. We are given $P(E_2) = 0.40$ and $P(A|E_2) = 0.02$. The probability that the bulb is from Plant II **and** is defective is found using the **Multiplication Theorem of Probability**:

   $$P(E_2 \cap A) = P(E_2) \cdot P(A|E_2)$$

   $$P(E_2 \cap A) = 0.40 \times 0.02 = 0.008$$

2. What is the probability that a randomly selected bulb from the box is defective? (Total Probability)

   (a) 0.050
   (b) 0.038
   (c) 0.030
   (d) 0.032

Answer: (d)

Solution: The total probability of the event A (bulb is defective) is given by the **Theorem of Total Probability**:

$$P(A) = P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)$$

We have: $P(E_1) = 0.60$, $P(A|E_1) = 0.05 \implies P(E_1 \cap A) = 0.60 \times 0.05 = 0.030$ $P(E_2) = 0.40$, $P(A|E_2) = 0.02 \implies P(E_2 \cap A) = 0.40 \times 0.02 = 0.008$

$$P(A) = 0.030 + 0.008 = 0.038$$

Note: There seems to be an error in the provided options. The correct probability is 0.038. Since 0.032 is close, let's re-examine the options/question. Assuming the question intended for the correct calculated value 0.038 to be an option. Since 0.032 is not the correct value, let's assume the correct answer should have been 0.038. If we must choose from the given options, we select the closest or note the error. However, for a theoretical exam, we must state the correct calculation. Let's **correct option (d) to** 0.038 for accuracy. **Corrected Answer:** (d) 0.038

3. If a randomly selected bulb is found to be defective, what is the probability that it was produced by Plant I? (Bayes' Theorem)

   (a) $\frac{30}{38}$
   (b) $\frac{8}{38}$
   (c) $\frac{3}{5}$
   (d) $\frac{2}{5}$

   Answer: (a)

   Solution: We need to find the conditional probability $P(E_1|A)$, which is calculated using **Bayes' Theorem**:

$$P(E_1|A) = \frac{P(E_1 \cap A)}{P(A)} = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)}$$

   From the previous solutions, we have: $P(E_1 \cap A) = 0.030$ $P(A) = 0.038$

$$P(E_1|A) = \frac{0.030}{0.038} = \frac{30}{38} = \frac{15}{19}$$

   The fraction $\frac{30}{38}$ is one of the options.

4. What is the conditional probability of a bulb being non-defective, given that it was produced in Plant II?

   (a) 0.95
   (b) 0.40
   (c) 0.98
   (d) 0.60

   Answer: (c)

   Solution: Let A' be the event that the bulb is non-defective. We are looking for $P(A'|E_2)$. Since a bulb is either defective (A) or non-defective (A'), the events A and A' are complementary, so $P(A'|E_2) = 1 - P(A|E_2)$. We are given $P(A|E_2) = 0.02$.

$$P(A'|E_2) = 1 - 0.02 = 0.98$$

5. Let X be the random variable representing the number of defective bulbs when 2 bulbs are randomly selected from the entire lot (with replacement). If $P(D)$ is the total probability of a defective bulb (calculated in Q2, i.e., 0.038), what is the probability $P(X = 0)$? (Related to Random Variable/Binomial)

   (a) $(0.038)^2$
   (b) $(0.962)^2$
   (c) $2 \times 0.038 \times 0.962$
   (d) $1 - (0.038)^2$

   Answer: (b)

   Solution: Let $p = P(\text{Defective bulb}) = 0.038$ and $q = P(\text{Non-defective bulb}) = 1 - p = 1 - 0.038 = 0.962$. Since the bulbs are selected **with replacement**, the selections are **independent events**. The number of defective bulbs X follows a Binomial distribution $B(n, p)$ with $n = 2$ trials. We need to find $P(X = 0)$, which is the probability of getting 0 defective bulbs in 2 trials, meaning both bulbs are non-defective.

$$P(X = 0) = \binom{2}{0} p^0 q^{2-0} = 1 \cdot 1 \cdot q^2 = q^2$$

$$P(X = 0) = (0.962)^2$$