Case Study 3: Matrix Operations in Real Life

Rohan is designing a robotic arm that must move in various directions based on coordinate instructions. To handle these instructions, he uses matrices to represent and process movement data. He performs addition, scalar multiplication, and matrix multiplication to determine the final position of the arm. During testing, he encounters situations where non-commutative multiplication leads to incorrect results if not carefully handled. To ensure safety and precision in his design, Rohan practices advanced operations on matrices and verifies results using matrix properties.

MCQ Questions:

1. What is the result of scalar multiplication $3 \times \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$?

   (a) $\begin{bmatrix} 6 & -3 \\ 0 & 12 \end{bmatrix}$

   (b) $\begin{bmatrix} 6 & 3 \\ 0 & 12 \end{bmatrix}$

   (c) $\begin{bmatrix} 5 & -2 \\ 0 & 7 \end{bmatrix}$

   (d) $\begin{bmatrix} 6 & -1 \\ 0 & 8 \end{bmatrix}$

   Answer: (a)
   Solution: Scalar multiplication means multiplying each element by 3: $3 \times 2 = 6$, $3 \times -1 = -3$, etc.

2. What is the sum of matrices $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$?

   (a) $\begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$

   (b) $\begin{bmatrix} 4 & 6 \\ 9 & 10 \end{bmatrix}$

   (c) $\begin{bmatrix} 7 & 6 \\ 8 & 7 \end{bmatrix}$

   (d) $\begin{bmatrix} 5 & 6 \\ 4 & 3 \end{bmatrix}$

   Answer: (a)
   Solution: Add corresponding elements: $1 + 5 = 6$, $2 + 6 = 8$, $3 + 7 = 10$, $4 + 8 = 12$.

3. Which operation is NOT always valid for any two matrices?

   (a) Scalar multiplication
   (b) Addition (if same order)
   (c) Transpose
   (d) Multiplication (any two matrices)

Answer: (d)
Solution: Multiplication of matrices is not always valid unless the number of columns in the first matrix equals the number of rows in the second.

4. The product of $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$ is:

   (a) $\begin{bmatrix} 7 & 5 \\ 3 & 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$

   (c) $\begin{bmatrix} 10 & 14 \\ 4 & 6 \end{bmatrix}$

   (d) $\begin{bmatrix} 6 & 8 \\ 3 & 4 \end{bmatrix}$

   Answer: (b)
   Solution: Multiplying by the identity matrix leaves the matrix unchanged.

5. If $A = \begin{bmatrix} 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, then the product AB is:

   (a) $\begin{bmatrix} 3 & 8 \end{bmatrix}$

   (b) $\begin{bmatrix} 11 \end{bmatrix}$

   (c) $\begin{bmatrix} 7 \end{bmatrix}$

   (d) Not defined

   Answer: (b)
   Solution: $AB = 1 \times 3 + 2 \times 4 = 3 + 8 = 11$, resulting in a 1×1 matrix.