Case Study 1

Priya, a Grade 12 student, was working on a project involving the motion of a particle along a straight line. She was given a velocity-time graph that was not accessible, but the velocity function was provided as $v(t) = 3t^2 + 2t + 1$. Her objective was to find the displacement function by integrating the velocity function. Her mathematics teacher explained that the displacement function is the antiderivative of the velocity function, and hence, Priya had to compute the indefinite integral of $v(t)$. She learned that indefinite integrals represent a family of curves differing only by a constant C, and geometrically, these curves are translations of each other. She used the formula $\int t^n \, dt = \frac{t^{n+1}}{n+1} + C$ and successfully derived the displacement function. Let's now help Priya solve related questions based on her project.

Standard Formulas Used:

- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
- $\int [af(x) + bg(x)] \, dx = a \int f(x) \, dx + b \int g(x) \, dx$
- If $F(x)$ is the integral of $f(x)$, then $\frac{d}{dx}[F(x)] = f(x)$

MCQ Questions:

1. What is the indefinite integral of the velocity function $v(t) = 3t^2 + 2t + 1$?

   (a) $t^3 + t^2 + t + C$
   (b) $t^3 + t^2 + C$
   (c) $t^3 + t^2 + 2t + C$
   (d) $t^3 + t^2 + 2t + 1 + C$

   Answer: (a)
   Solution:

   $$\int (3t^2 + 2t + 1) \, dt = 3 \cdot \frac{t^3}{3} + 2 \cdot \frac{t^2}{2} + t + C = t^3 + t^2 + t + C$$

2. What is the geometrical meaning of the constant of integration C in Priya's displacement function?

   (a) It represents acceleration.
   (b) It represents initial displacement.
   (c) It represents velocity.
   (d) It represents time.

   Answer: (b)
   Solution: The constant C represents the value of the function at a specific point — in this case, initial displacement.

3. Which of the following properties is used in integrating $v(t)$ term-wise?

   (a) Substitution method
   (b) Linearity of integration
   (c) Integration by parts

(d) Trigonometric substitution

Answer: (b)
Solution: The linearity property allows integrating each term separately.

4. If $s(t)$ is the displacement function found as $s(t) = t^3 + t^2 + t + C$, then what is $\frac{ds}{dt}$?

   (a) $3t^2 + 2t + 1$
   (b) $t^2 + t + 1$
   (c) $3t + 2$
   (d) $3t^2 + t + 1$

   Answer: (a)
   Solution: Differentiating $s(t)$ gives $s'(t) = 3t^2 + 2t + 1$ which is the original velocity function.

5. Which of the following integrals is equal to $t^3 + t^2 + t + C$?

   (a) $\int (t^2 + t + 1)\, dt$
   (b) $\int (3t^2 + 2t + 1)\, dt$
   (c) $\int (3t + 2 + 1)\, dt$
   (d) $\int (t^3 + t^2 + t)\, dt$

   Answer: (b)
   Solution: As shown earlier, integrating $3t^2 + 2t + 1$ gives $t^3 + t^2 + t + C$.