Case Study 4: Applying Integration by Parts in Practical Scenarios

Ritika, a student of Class 12, was working on finding the integral of functions that are products of algebraic and transcendental terms, such as $\int x \ln x \, dx$. Her teacher introduced her to the method of **Integration by Parts**, which is used when integrals involve a product of two functions. She learned the formula:

$$\int u \cdot v \, dx = u \int v \, dx - \int \left(\frac{du}{dx} \int v \, dx \right) dx$$

To apply the rule, Ritika used the **LIATE** rule (Logarithmic, Inverse, Algebraic, Trigonometric, Exponential) to choose u and v. She let $u = \ln x$ and $dv = x \, dx$ while solving $\int x \ln x \, dx$, and got the result $\frac{x^2}{2} \ln x - \frac{x^2}{4} + C$. With practice, she was able to handle various types of problems using this method effectively. Let's test some cases based on her learning.

Key Concepts and Formula:

- $\int u \cdot v \, dx = u \int v \, dx - \int \left(\frac{du}{dx} \int v \, dx \right) dx$
- Use the LIATE rule to choose u
- Examples involve products like $x \ln x$, xe^x, $x \sin x$, etc.

MCQ Questions:

1. What is the value of $\int x \ln x \, dx$?

   (a) $\frac{x^2}{2} \ln x + \frac{x^2}{4} + C$
   (b) $\frac{x^2}{2} \ln x - \frac{x^2}{4} + C$
   (c) $x \ln x - x + C$
   (d) $\frac{x^2}{2} \ln x + C$

   Answer: (b)
   Solution: Using integration by parts:

$$u = \ln x, \quad dv = x \, dx \Rightarrow du = \frac{1}{x} dx, \quad v = \frac{x^2}{2}$$

$$\int x \ln x \, dx = \ln x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} \, dx = \frac{x^2}{2} \ln x - \int \frac{x}{2} \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

2. Choose the correct u and dv using LIATE for $\int xe^x \, dx$.

   (a) $u = e^x$, $dv = x \, dx$
   (b) $u = x$, $dv = e^x \, dx$
   (c) $u = x$, $dv = dx$
   (d) $u = e^x$, $dv = dx$

   Answer: (b)
   Solution: According to LIATE, Algebraic (x) comes before Exponential (e^x), so $u = x$ and $dv = e^x \, dx$

3. Find the value of $\int xe^x\,dx$.

   (a) $xe^x - e^x + C$
   (b) $xe^x + e^x + C$
   (c) $x^2e^x + C$
   (d) $e^x + C$

   Answer: (a)
   Solution:

$$u = x, \quad dv = e^x\,dx \Rightarrow du = dx, \quad v = e^x$$
$$\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x + C$$

4. Evaluate $\int x\cos x\,dx$.

   (a) $x\sin x + \cos x + C$
   (b) $x\cos x + \sin x + C$
   (c) $x\sin x + \cos x + C$
   (d) $x\sin x + \cos x + C$

   Answer: (a)
   Solution:

$$u = x, \quad dv = \cos x\,dx \Rightarrow du = dx, \quad v = \sin x$$
$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx = x\sin x + \cos x + C$$

5. What is $\int \ln x\,dx$ using integration by parts?

   (a) $x\ln x + x + C$
   (b) $x\ln x - x + C$
   (c) $\frac{1}{x} + C$
   (d) $\ln^2 x + C$

   Answer: (b)
   Solution:

$$\int \ln x\,dx = x\ln x - \int x\,\frac{1}{x}\,dx = x\ln x - x + C$$