Case Study 3: Mapping Relations in a Library System

A library system is trying to keep track of which books are borrowed by which students in Grade 12. Let the set of students be $S = \{s_1, s_2, s_3, s_4\}$ and the set of books be $B = \{b_1, b_2, b_3\}$. A relation R is defined from S to B by the ordered pairs $\{(s_1, b_1), (s_1, b_2), (s_2, b_3), (s_3, b_2), (s_4, b_1), (s_4, b_3)\}$. Based on this scenario, answer the following questions:

1. What is the domain of the relation R?

   (a) $\{b_1, b_2, b_3\}$
   (b) $\{s_1, s_2, s_3, s_4\}$
   (c) $\{s_1, s_2, s_3\}$
   (d) $\{s_1, s_2, b_1, b_2\}$

   Answer: (b) $\{s_1, s_2, s_3, s_4\}$
   Solution: The domain of the relation is the set of first elements in the ordered pairs, which are the students.

2. What is the range of the relation R?

   (a) $\{b_1, b_2, b_3\}$
   (b) $\{s_1, s_2, s_3, s_4\}$
   (c) $\{s_1, s_2, b_1, b_2\}$
   (d) $\{b_1, b_2\}$

   Answer: (a) $\{b_1, b_2, b_3\}$
   Solution: The range is the set of second elements (books) in the ordered pairs.

3. Determine the codomain of the relation R.

   (a) $\{b_1, b_2, b_3\}$
   (b) $\{s_1, s_2, s_3, s_4\}$
   (c) $\{s_1, s_2, b_1, b_2\}$
   (d) $\{b_1, b_2\}$

   Answer: (a) $\{b_1, b_2, b_3\}$
   Solution: The codomain of the relation is the set of books, as R is defined from students to books.

4. How many ordered pairs are there in the relation R?

   (a) 4
   (b) 6
   (c) 5
   (d) 3

   Answer: (b) 6
   Solution: The relation R consists of 6 ordered pairs.

5. Is the relation R a function from S to B?

   (a) Yes
   (b) No

(c) Only if S has more elements

(d) Cannot be determined

Answer: (b) No

Solution: For R to be a function, each student (element in S) should be associated with exactly one book. Here, s_1 and s_4 are each associated with two books, so R is not a function.