Case Study 4: Understanding Matrix Properties in Data Science

In a data science project, Ananya is analyzing customer behavior using matrices. She uses symmetric and skew-symmetric matrices to represent user preference data. Symmetric matrices help her understand mutual ratings, while skew-symmetric matrices highlight contrasts. Additionally, she employs transpose operations to change perspectives between product-based and user-based views. By multiplying matrices, she combines different datasets for analysis. However, she learns that matrix multiplication is not commutative, meaning $AB \neq BA$, which affects her model accuracy. She practices matrix operations to ensure reliable analytics.

MCQ Questions:

1. Which of the following is a symmetric matrix?

   (a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

   (b) $\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

   Answer: (b)
   Solution: A matrix is symmetric if it equals its transpose. Option (b) satisfies this property.

2. What is the transpose of the matrix $A = \begin{bmatrix} 4 & -2 \\ 1 & 3 \end{bmatrix}$?

   (a) $\begin{bmatrix} 4 & 1 \\ -2 & 3 \end{bmatrix}$

   (b) $\begin{bmatrix} 4 & 1 \\ 3 & -2 \end{bmatrix}$

   (c) $\begin{bmatrix} 4 & -2 \\ 1 & 3 \end{bmatrix}$

   (d) $\begin{bmatrix} -2 & 4 \\ 3 & 1 \end{bmatrix}$

   Answer: (a)
   Solution: Transpose is found by switching rows and columns.

3. Which matrix is skew-symmetric?

   (a) $\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 5 & -2 \\ -2 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

Answer: (a)
Solution: A matrix is skew-symmetric if $A^T = -A$. Option (a) satisfies this.

4. Which statement is true about matrix multiplication?

   (a) Always commutative: $AB = BA$
   (b) Always associative: $(AB)C = A(BC)$
   (c) Only possible for square matrices
   (d) Only valid for matrices of the same order

   Answer: (b)
   Solution: Matrix multiplication is associative but not necessarily commutative.

5. If $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$, what is A^T?

   (a) $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$
   (b) $\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$
   (c) $\begin{bmatrix} 0 & 0 \\ 2 & -2 \end{bmatrix}$
   (d) $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$

   Answer: (a)
   Solution: Switching rows and columns gives the transpose: first row becomes first column, etc.