Case Study 4: Quality Control in a Toy Factory (Probability - Random Variable)

Case Study Description:

A toy factory manufactures a specific small electronic component in batches. Historically, the factory has an average defect rate of 10% for this component. The quality control manager decides to implement a detailed inspection process. During this process, inspectors are instructed to randomly select 4 components from a batch and test them for defects. The number of defective components found in the sample of 4 is the **Random Variable** X. The factory manager is interested in analyzing the distribution of X, specifically the probability of finding 0, 1, 2, 3, or 4 defective components. This scenario represents a classic application of the **Binomial Distribution** since the trials (testing each component) are independent, there are a fixed number of trials ($n = 4$), and the probability of a success (finding a defective component, $p = 0.10$) remains constant. Understanding the **probability distribution** is vital for setting acceptable quality benchmarks. The manager also needs to calculate the **mean** (expected number of defects) and the **variance** of this random variable X to predict long-term quality trends and assess risk. For this analysis, we assume the batch size is very large, making the selection of components approximately independent, which satisfies the conditions for a Binomial distribution.

Let X be the random variable representing the number of defective components out of a sample of $n = 4$. The probability of a defective component (success) is $p = 0.10$. The probability of a non-defective component (failure) is $q = 0.90$.

MCQ Questions (5 Questions)

1. What is the probability of finding exactly **two** defective components in the sample of 4? (Probability Distribution $P(X = 2)$)

   (a) $4 \times (0.10)^2 \times (0.90)^2$
   (b) $6 \times (0.10)^2 \times (0.90)^2$
   (c) $1 \times (0.10)^2 \times (0.90)^2$
   (d) $4 \times (0.10)^2 \times (0.90)^3$

   Answer: (b)

   Solution: The random variable X follows a Binomial distribution, $P(X = k) = \binom{n}{k} p^k q^{n-k}$. Here $n = 4$, $k = 2$, $p = 0.10$, $q = 0.90$.

$$P(X = 2) = \binom{4}{2}(0.10)^2(0.90)^{4-2}$$

$$\binom{4}{2} = \frac{4!}{2!2!} = 6$$

$$P(X = 2) = 6 \times (0.10)^2 \times (0.90)^2$$

2. What is the probability of finding **no** defective components in the sample of 4? (Probability Distribution $P(X = 0)$)

   (a) 0.90
   (b) $(0.90)^4$
   (c) $1 - (0.10)^4$

(d) $4 \times 0.10 \times 0.90$

Answer: (b)

Solution: We need to find $P(X = 0)$.

$$P(X = 0) = \binom{4}{0} p^0 q^4$$

$$\binom{4}{0} = 1$$

$$P(X = 0) = 1 \times (0.10)^0 \times (0.90)^4 = (0.90)^4$$

3. What is the **mean** (Expected Value) of the random variable X (the expected number of defective components in a sample of 4)? (Mean of a Random Variable)

   (a) 0.4
   (b) 0.1
   (c) 4.0
   (d) 0.36

   Answer: (a)

   Solution: For a Binomial distribution $B(n, p)$, the **Mean** or Expected Value is $E(X) = n \cdot p$. We have $n = 4$ and $p = 0.10$.

   $$E(X) = 4 \times 0.10 = 0.4$$

   The expected number of defects is 0.4 per sample of 4.

4. What is the **variance** of the random variable X? (Variance of a Random Variable)

   (a) 0.40
   (b) 0.04
   (c) 0.36
   (d) 0.90

   Answer: (c)

   Solution: For a Binomial distribution $B(n, p)$, the **Variance** is $\mathrm{Var}(X) = n \cdot p \cdot q$. We have $n = 4$, $p = 0.10$, and $q = 0.90$.

   $$\mathrm{Var}(X) = 4 \times 0.10 \times 0.90$$

   $$\mathrm{Var}(X) = 0.4 \times 0.90 = 0.36$$

5. What is the probability that there is **at least one** defective component in the sample of 4? (Complementary Event)

   (a) $1 - (0.10)^4$
   (b) $1 - (0.90)^4$
   (c) $1 - 0.4$

(d) $1 - 0.36$

Answer: (b)

Solution: The event "at least one defective component" is $P(X \geq 1)$. The complementary event is $P(X = 0)$, which is "no defective components".

$$P(X \geq 1) = 1 - P(X = 0)$$

From Q2, $P(X = 0) = (0.90)^4$.

$$P(X \geq 1) = 1 - (0.90)^4$$