Case Study 2: Algebraic Operations on Vectors

A group of engineering students are working on a robotics project that involves controlling a robot's path using vector inputs. They define the robot's movement in terms of vector displacements. The path consists of sequential displacements represented as vectors. They use vector addition to compute the resultant path, scalar multiplication to change the speed (magnitude) of the movement, and subtraction to find the change in direction. The team learns that the Triangle Law of vector addition helps combine vectors when applied head-to-tail, while the Parallelogram Law is useful when vectors are applied from the same point. They apply vector algebra in two and three dimensions to program the robot's navigation efficiently.

Theory and Formulae Related to Algebra of Vectors:

- Vector addition: $\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$
- Vector subtraction: $\vec{a} - \vec{b} = (a_1 - b_1)\hat{i} + (a_2 - b_2)\hat{j} + (a_3 - b_3)\hat{k}$
- Scalar multiplication: $k\vec{a} = (ka_1)\hat{i} + (ka_2)\hat{j} + (ka_3)\hat{k}$
- Triangle Law: $\vec{a} + \vec{b}$ is the third side of triangle formed when vectors are placed head to tail
- Parallelogram Law: Resultant of $\vec{a}$ and $\vec{b}$ is the diagonal of the parallelogram formed by $\vec{a}$ and $\vec{b}$ from same initial point

MCQ Questions

1. If $\vec{a} = 2\hat{i} + 3\hat{j}$ and $\vec{b} = \hat{i} - 2\hat{j}$, find $\vec{a} + \vec{b}$.

   (a) $3\hat{i} + \hat{j}$

   (b) $\hat{i} + \hat{j}$

   (c) $\hat{i} + 5\hat{j}$

   (d) $3\hat{i} - \hat{j}$

   Answer: (a)
   Solution: $\vec{a} + \vec{b} = (2 + 1)\hat{i} + (3 + (-2))\hat{j} = 3\hat{i} + \hat{j}$

2. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, find $2\vec{a}$.

   (a) $\hat{i} + 4\hat{j} + 6\hat{k}$

   (b) $2\hat{i} + 2\hat{j} + 6\hat{k}$

   (c) $2\hat{i} + 4\hat{j} + 6\hat{k}$

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(d) $3\hat{i} + 4\hat{j} + 3\hat{k}$

Answer: (c)
Solution: $2\vec{a} = 2\hat{i} + 4\hat{j} + 6\hat{k}$

3. If $\vec{a} = 4\hat{i} - 2\hat{j}$ and $\vec{b} = -\hat{i} + 5\hat{j}$, then $\vec{a} - \vec{b}$ equals:

   (a) $3\hat{i} - 7\hat{j}$

   (b) $5\hat{i} + 3\hat{j}$

   (c) $-5\hat{i} + 7\hat{j}$

   (d) $4\hat{i} + 3\hat{j}$

   Answer: (a)
   Solution: $\vec{a} - \vec{b} = (4 - (-1))\hat{i} + (-2 - 5)\hat{j} = 5\hat{i} - 7\hat{j}$

4. What is the resultant vector when $\vec{a}$ and $\vec{b}$ are represented as two adjacent sides of a parallelogram?

   (a) $\vec{a} - \vec{b}$

   (b) $\vec{a} + \vec{b}$

   (c) $\vec{b} - \vec{a}$

   (d) $\vec{a} \times \vec{b}$

   Answer: (b)
   Solution: In the Parallelogram Law of vector addition, the diagonal represents $\vec{a} + \vec{b}$

5. If two vectors $\vec{a}$ and $\vec{b}$ are such that $\vec{a} + \vec{b} = \vec{0}$, what can we say?

   (a) Vectors are collinear

   (b) Vectors are perpendicular

   (c) Vectors are equal in magnitude and opposite in direction

   (d) Vectors are unit vectors

   Answer: (c)
   Solution: If $\vec{a} + \vec{b} = \vec{0}$, then $\vec{a} = -\vec{b}$. Hence, they are equal in magnitude and opposite in direction.

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