Case Study 4: Dot Product and Its Applications

In a mechanics lab, a team is analyzing the motion of a block being pushed along an inclined plane. The force applied and the displacement are represented by vectors. To calculate the work done, the students use the concept of dot product. They also explore how the dot product helps find the angle between two vectors and project one vector onto another. The dot product is useful not only in physics but also in computer graphics, engineering, and navigation systems, where direction and magnitude both play a role. By applying formulas, the students calculate work done, identify orthogonal vectors, and understand geometric interpretations of scalar product.

Theory and Formulae Related to Dot Product:

- Dot product: $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$
- Alternate form: $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$
- Angle between vectors: $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$
- If $\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a} \perp \vec{b}$
- Projection of $\vec{a}$ on $\vec{b}$: $\text{Proj}_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

MCQ Questions

1. If $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$, find $\vec{a} \cdot \vec{b}$

   (a) 6

   (b) 4

   (c) 2

   (d) 8

   Answer: (b)
   Solution: $\vec{a} \cdot \vec{b} = 3(1) + (-2)(2) + 1(3) = 3 - 4 + 3 = 2$

2. Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular. Which condition is true?

   (a) $\vec{a} \cdot \vec{b} = 1$

   (b) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$

   (c) $\vec{a} \cdot \vec{b} = 0$

   (d) $\vec{a} \cdot \vec{b} = -1$

   Answer: (c)
   Solution: If vectors are perpendicular, $\theta = 90^\circ \Rightarrow \cos\theta = 0 \Rightarrow \vec{a} \cdot \vec{b} = 0$

3. If $|\vec{a}| = 5$, $|\vec{b}| = 6$, and angle between them is $60°$, find $\vec{a} \cdot \vec{b}$

   (a) 10

   (b) 15

   (c) 30

   (d) None of these

   Answer: (c)
   Solution: $\vec{a} \cdot \vec{b} = 5 \cdot 6 \cdot \cos 60° = 30 \cdot \frac{1}{2} = 15$

4. If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath}$ and $\vec{b} = \hat{\imath} - \hat{\jmath}$, what is the angle between them?

   (a) $45°$

   (b) $60°$

   (c) $90°$

   (d) $120°$

   Answer: (c)
   Solution:

$$\vec{a} \cdot \vec{b} = 2(1) + 2(-1) = 2 - 2 = 0$$
$$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90°$$

5. The projection of $\vec{a} = 3\hat{\imath} + 4\hat{\jmath}$ on $\vec{b} = 5\hat{\imath}$ is:

   (a) 3

   (b) 5

   (c) 4

   (d) 2

   Answer: (a)
   Solution:

$$\vec{a} \cdot \vec{b} = 3 \cdot 5 + 4 \cdot 0 = 15$$
$$|\vec{b}| = 5$$
$$\text{Projection} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{15}{5} = 3$$