

## Case Study 2: Analyzing Friendships in a Classroom

In a class of 12th-grade students, the teacher wants to examine the friendship patterns among the students to understand social dynamics. Let the set of boys in the class be  $B = \{b_1, b_2, b_3\}$  and the set of girls be  $G = \{g_1, g_2, g_3, g_4\}$ . A relation  $F$  is defined from  $B$  to  $G$ , where  $(b_i, g_j)$  means boy  $b_i$  considers girl  $g_j$  as a friend. The relation is given by the set  $\{(b_1, g_1), (b_1, g_2), (b_2, g_2), (b_2, g_3), (b_3, g_4)\}$ . Based on this, answer the following multiple-choice questions:

1. What is the domain of the relation  $F$ ?

- (a)  $\{b_1, b_2, b_3\}$
- (b)  $\{g_1, g_2, g_3, g_4\}$
- (c)  $\{b_1, b_2\}$
- (d)  $\{b_1, b_2, b_3, g_1, g_2, g_3, g_4\}$

**Answer:** (a)  $\{b_1, b_2, b_3\}$

**Solution:** The domain of the relation is the set of first elements (boys) in each pair.

2. What is the range of the relation  $F$ ?

- (a)  $\{b_1, b_2, b_3\}$
- (b)  $\{g_1, g_2, g_3, g_4\}$
- (c)  $\{g_1, g_2, g_3, g_4\}$
- (d)  $\{b_1, g_1, g_2, g_3, g_4\}$

**Answer:** (c)  $\{g_1, g_2, g_3, g_4\}$

**Solution:** The range is the set of second elements (girls) in the ordered pairs.

3. Determine the codomain of the relation  $F$ .

- (a)  $\{b_1, b_2, b_3\}$
- (b)  $\{g_1, g_2, g_3, g_4\}$
- (c)  $\{g_1, g_2, g_3\}$
- (d)  $\{b_1, b_2, b_3, g_1, g_2, g_3, g_4\}$

**Answer:** (b)  $\{g_1, g_2, g_3, g_4\}$

**Solution:** The codomain is the set of girls since  $F$  is defined from boys to girls.

4. How many elements does the relation  $F$  have?

- (a) 4
- (b) 5
- (c) 6
- (d) 3

**Answer:** (b) 5

**Solution:** The relation consists of 5 ordered pairs.

5. Is the relation  $F$  a function from  $B$  to  $G$ ?

- (a) Yes
- (b) No

- (c) Cannot be determined
- (d) Only if  $B$  has more elements

**Answer:** (b) No

**Solution:** For  $F$  to be a function, each element of the domain  $B$  must be paired with exactly one element in  $G$ . Here,  $b_1$  and  $b_2$  are each paired with two different girls, so  $F$  is not a function.