

Case Study 5: Analyzing a Relation in a Classroom Setting

In a classroom of five students, the teacher defines a relation R on the set $S = \{A, B, C, D, E\}$. The relation R is described as follows: A student x is related to student y if x and y have exactly the same number of friends within the classroom. The relation R is defined as $\{(A, A), (B, B), (C, C), (D, D), (E, E), (A, B), (B, A), (C, D), (D, C)\}$. Analyze this scenario and answer the following questions:

1. Is the relation R reflexive?

- (a) Yes
- (b) No
- (c) Only for some students
- (d) Cannot be determined

Answer: (a) Yes

Solution: Every element of S is related to itself, as $(A, A), (B, B), (C, C), (D, D), (E, E)$ are all in R . Hence, R is reflexive.

2. Is the relation R symmetric?

- (a) Yes
- (b) No
- (c) Only for some students
- (d) Cannot be determined

Answer: (a) Yes

Solution: For any (x, y) in R , (y, x) is also in R , as seen with pairs like (A, B) and (B, A) , (C, D) and (D, C) . Therefore, R is symmetric.

3. Is the relation R transitive?

- (a) Yes
- (b) No
- (c) Only for some pairs
- (d) Cannot be determined

Answer: (b) No

Solution: For example, (A, B) and (B, A) exist, but there is no pair like (A, A) (except the reflexive one), so transitivity does not extend beyond direct pairs.

4. Determine whether the relation R is an equivalence relation.

- (a) Yes
- (b) No
- (c) Only if transitive
- (d) Cannot be determined

Answer: (b) No

Solution: An equivalence relation must be reflexive, symmetric, and transitive. Here, R is not transitive, so it is not an equivalence relation.

5. How many ordered pairs are in the relation R ?

- (a) 9
- (b) 10
- (c) 8
- (d) 7

Answer: (a) 9

Solution: Counting all ordered pairs: $(A, A), (B, B), (C, C), (D, D), (E, E), (A, B), (B, A), (C, D), (D, C)$.
This totals 9 pairs.