

Case Study 4: Relations in a Social Media Platform

Consider a social media platform that connects people through friend requests. Let the set of users be $U = \{u_1, u_2, u_3, u_4, u_5\}$. The relation F on U is defined by the ordered pairs $\{(u_1, u_2), (u_2, u_1), (u_2, u_3), (u_3, u_2), (u_4, u_5), (u_5, u_4)\}$, where (u_i, u_j) means u_i sent a friend request to u_j . Based on this scenario, answer the following questions:

1. Determine if the relation F is symmetric.

- (a) Yes
- (b) No
- (c) Sometimes
- (d) Cannot be determined

Answer: (a) Yes

Solution: A relation is symmetric if (a, b) in F implies (b, a) is also in F . All pairs have their reverses, so F is symmetric.

2. Is the relation F reflexive?

- (a) Yes
- (b) No
- (c) Only for some users
- (d) Cannot be determined

Answer: (b) No

Solution: Reflexivity requires (u, u) for all u in U , which is not the case here.

3. Is the relation F transitive?

- (a) Yes
- (b) No
- (c) Only for some users
- (d) Cannot be determined

Answer: (b) No

Solution: For example, (u_1, u_2) and (u_2, u_3) exist, but (u_1, u_3) does not. Hence, F is not transitive.

4. Is the relation F an equivalence relation?

- (a) Yes
- (b) No
- (c) Only if reflexive
- (d) Cannot be determined

Answer: (b) No

Solution: For a relation to be an equivalence relation, it must be reflexive, symmetric, and transitive. Since F is not reflexive or transitive, it is not an equivalence relation.

5. How many symmetric pairs exist in the relation F ?

- (a) 6

- (b) 4
- (c) 3
- (d) 2

Answer: (b) 4

Solution: The pairs (u_1, u_2) and (u_2, u_1) form one symmetric pair, (u_2, u_3) and (u_3, u_2) another, (u_4, u_5) and (u_5, u_4) another, totaling 4 symmetric pairs.