

Case Study 4: Temperature Monitoring – Increasing and Decreasing Functions in Daily Life

A weather monitoring station collects data of temperature variations during the day and models the temperature function as $T(t) = -2t^2 + 12t + 5$, where t is the time in hours after 6 AM and $T(t)$ is the temperature in degrees Celsius. To find out when the temperature is rising or falling, meteorologists analyze the derivative of the function to determine intervals of increase and decrease. Understanding these trends helps farmers, commuters, and businesses plan activities like irrigation, transportation, and energy consumption efficiently. The concept of increasing and decreasing functions from calculus provides critical insights into such natural phenomena.

MCQ Questions:

1. What is the first derivative $T'(t)$ of the function $T(t) = -2t^2 + 12t + 5$?

(a) $T'(t) = -4t + 12$

(b) $T'(t) = -4t - 12$

(c) $T'(t) = 4t + 12$

(d) $T'(t) = 2t - 12$

Answer: (A) $T'(t) = -4t + 12$

Solution: Derivative of $T(t) = -2t^2 + 12t + 5$ is $T'(t) = -4t + 12$

2. For what value of t is the temperature neither increasing nor decreasing?

(a) $t = 3$

(b) $t = 6$

(c) $t = 4$

(d) $t = 12$

Answer: (A) $t = 3$

Solution: Set $T'(t) = 0 \Rightarrow -4t + 12 = 0 \Rightarrow t = 3$

3. In which interval is the temperature increasing?

(a) $(0, 3)$

(b) $(3, 6)$

(c) $(3, \infty)$

(d) $(0, \infty)$

Answer: (A) $(0, 3)$

Solution: $T'(t) = -4t + 12 > 0$ when $t < 3$, so the function is increasing in $(0, 3)$

4. In which interval is the temperature decreasing?

- (a) $(0, 3)$
- (b) $(3, \infty)$
- (c) $(-\infty, 3)$
- (d) $(0, \infty)$

Answer: (B) $(3, \infty)$

Solution: $T'(t) < 0$ when $t > 3$, so the function is decreasing in $(3, \infty)$

5. What is the maximum temperature attained and at what time?

- (a) $T(3) = 23$ at 9 AM
- (b) $T(3) = 20$ at 9 AM
- (c) $T(3) = 22$ at 9 AM
- (d) $T(3) = 23$ at 6 AM

Answer: (A) $T(3) = 23$ at 9 AM

Solution: $t = 3 \Rightarrow T(3) = -2(3)^2 + 12(3) + 5 = -18 + 36 + 5 = 23$ at 6 AM + 3 hr = 9 AM