# Case Study 5: Student Performance in Two Subjects (Probability - Independent Events)

**Case Study Description:**

In a Grade 12 class, student performance in Mathematics and Physics is being analyzed. The school principal considers passing in Mathematics ($M$) and passing in Physics ($P$) as **independent events** for any randomly chosen student. The probability that a student passes in Mathematics is 0.80, and the probability that a student passes in Physics is 0.70. The principal wants to calculate various probabilities related to a student's performance in both subjects. Since the events are independent, the **Multiplication Theorem of Probability** for independent events simplifies the calculation of the probability of passing both or failing both. This assumption of independence means that a student's preparation or aptitude in one subject does not influence their outcome in the other subject. While this is often a simplification in real life, it serves as a powerful model for understanding the basic rules of probability.

Let $M$ be the event of passing Mathematics, and $P$ be the event of passing Physics. We are given $P(M) = 0.80$ and $P(P) = 0.70$. The events $M$ and $P$ are independent. The goal is to calculate the probabilities of compound events, such as passing at least one subject, and failing both, and to test the conceptual understanding of conditional probability for independent events.

## MCQ Questions (5 Questions)

1. What is the probability that a randomly selected student passes in **both** Mathematics and Physics? (Multiplication Theorem for Independent Events)

   (a) 0.56
   (b) 1.50
   (c) 0.14
   (d) 0.24

   **Answer:** (a)

   **Solution:** Since $M$ and $P$ are independent events, the probability of both occurring is:

   $$P(M \cap P) = P(M) \cdot P(P)$$

   $$P(M \cap P) = 0.80 \times 0.70 = 0.56$$

2. What is the probability that a randomly selected student **fails** in both Mathematics and Physics? (Independent Events)

   (a) 0.14
   (b) 0.06
   (c) 0.20
   (d) 0.94

   **Answer:** (b)

   **Solution:** Let $M'$ be the event of failing Mathematics and $P'$ be the event of failing Physics. $P(M') = 1 - P(M) = 1 - 0.80 = 0.20$ $P(P') = 1 - P(P) = 1 - 0.70 = 0.30$ Since $M$ and $P$ are independent, $M'$ and $P'$ are also independent.

   $$P(M' \cap P') = P(M') \cdot P(P')$$

$$P(M' \cap P') = 0.20 \times 0.30 = 0.06$$

3. What is the probability that a randomly selected student passes in **at least one** of the two subjects? (Union of Events)

   (a) 0.70
   (b) 0.80
   (c) 0.94
   (d) 0.44

   **Answer:** (c)

   **Solution:** The event "passing at least one subject" is $P(M \cup P)$. Using the formula for the union of two events:

   $$P(M \cup P) = P(M) + P(P) - P(M \cap P)$$

   Using the result from Q1, $P(M \cap P) = 0.56$.

   $$P(M \cup P) = 0.80 + 0.70 - 0.56$$

   $$P(M \cup P) = 1.50 - 0.56 = 0.94$$

   Alternatively, using the complementary event (failing both, from Q2):

   $$P(M \cup P) = 1 - P(M' \cap P') = 1 - 0.06 = 0.94$$

4. What is the probability that the student passes in Mathematics, **given** that they have passed in Physics? (Conditional Probability for Independent Events)

   (a) 0.56
   (b) 0.70
   (c) 0.80
   (d) 0.24

   **Answer:** (c)

   **Solution:** We need to find $P(M|P)$. Since the events $M$ and $P$ are **independent**, the occurrence of $P$ does not affect the probability of $M$.

   $$P(M|P) = P(M)$$

   $$P(M|P) = 0.80$$

5. Let $X$ be the random variable representing the number of subjects passed (out of 2). If a student passes in exactly one subject, what is the value of the probability $P(X = 1)$? (Probability Distribution)

   (a) $0.24 + 0.14$
   (b) 0.56
   (c) $0.80 + 0.70$
   (d) 0.06

**Answer:** (a)

**Solution:** The event "passes in exactly one subject" is the union of two mutually exclusive possibilities:

(a) Passes Math and Fails Physics ($M \cap P'$).

(b) Fails Math and Passes Physics ($M' \cap P$).

$$P(M \cap P') = P(M) \cdot P(P') = 0.80 \times 0.30 = 0.24$$

$$P(M' \cap P) = P(M') \cdot P(P) = 0.20 \times 0.70 = 0.14$$

$$P(X = 1) = P(M \cap P') + P(M' \cap P) = 0.24 + 0.14 = 0.38$$

The option $0.24 + 0.14$ correctly represents the calculation required.