Case Study Based Questions
Chapter: Inverse Trigonometric Functions

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Case Study 2:
Anita is working on a class assignment focused on the behavior of inverse trigonometric functions and how they apply to real-world angles and identities. Her teacher assigns her a task to simplify expressions using inverse trigonometric identities and to correctly identify principal values from given scenarios. Anita explores how the inverse functions of sine, cosine, tangent, and others differ in their domain, range, and graphical properties. She realizes that a deep understanding of the principal value branch helps avoid errors when evaluating inverse expressions. While solving problems, she uses properties like $\sin^{-1} x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ and symmetry relations such as $\cos^{-1}(-x) = \pi - \cos^{-1} x$. The following questions are based on Anita's learning experience.

MCQ Questions:

1. The expression $\cos^{-1}(-\frac{1}{2})$ simplifies to:

   (a) $\frac{\pi}{3}$
   (b) $\frac{2\pi}{3}$
   (c) $\frac{5\pi}{6}$
   (d) $\frac{3\pi}{4}$

   Answer: (b)
   Solution: $\cos^{-1}(-x) = \pi - \cos^{-1} x$. So, $\cos^{-1}(-\frac{1}{2}) = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

2. Which of the following is true for all $x \in (-1, 1)$?

   (a) $\sin^{-1} x = \cos^{-1}(x)$
   (b) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
   (c) $\cos^{-1} x - \sin^{-1} x = \frac{\pi}{2}$
   (d) $\sin^{-1} x + \cos^{-1} x = \pi$

   Answer: (b)
   Solution: For $x \in [-1, 1]$, $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ is a well-known identity in inverse trigonometry.

3. The expression $\sin^{-1}\left(\frac{3}{5}\right)$ can also be written as:

(a) $\tan^{-1}\left(\frac{3}{4}\right)$

(b) $\cos^{-1}\left(\frac{4}{5}\right)$

(c) $\tan^{-1}\left(\frac{3}{\sqrt{16}}\right)$

(d) All of the above

Answer: (d)
Solution: If $\sin^{-1}\left(\frac{3}{5}\right) = \theta$, then $\sin\theta = \frac{3}{5}$, so $\cos\theta = \frac{4}{5}$, and $\tan\theta = \frac{3}{4}$. Thus, $\theta = \tan^{-1}\left(\frac{3}{4}\right) = \cos^{-1}\left(\frac{4}{5}\right)$.

4. The range of $\tan^{-1} x$ is:

(a) $[0, \pi]$

(b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(c) $[0, \frac{\pi}{2})$

(d) $(-\pi, \pi)$

Answer: (b)
Solution: The principal value branch of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, which excludes the endpoints.

5. The value of $\cot^{-1}(\sqrt{3})$ is:

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{2}$

Answer: (b)
Solution: $\cot^{-1}(\sqrt{3}) = \frac{\pi}{6}$ because $\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$ and $\frac{\pi}{6} \in (0, \pi)$, which is the principal value branch for $\cot^{-1} x$.