Case Study 5:
In a robotics workshop, a group of students is learning how robotic arms use angles and trigonometry to determine position and movement. One scenario involves determining the joint angle using inverse trigonometric functions when the coordinates of the end-effector (robotic hand) are known. For instance, to compute the angle made by the arm with a horizontal axis, students use $\theta = \tan^{-1}(y/x)$. They also learn that for certain positions, the angle calculated must lie within the principal value branch to ensure the correct interpretation by the robot's controller. The trainer emphasizes the need for understanding domain restrictions and composite identities like $\cos^{-1}(\cos x)$ and their limitations. The following questions assess conceptual clarity based on this workshop.

MCQ Questions:

1. The value of $\tan^{-1}(-x)$ is:

   (a) $\tan^{-1} x$
   (b) $-\tan^{-1} x$
   (c) $\frac{\pi}{2} - \tan^{-1} x$
   (d) $\pi - \tan^{-1} x$

   Answer: (b)
   Solution: $\tan^{-1} x$ is an odd function. Therefore, $\tan^{-1}(-x) = -\tan^{-1} x$.

2. The principal value of $\cos^{-1}(\cos 3\pi)$ is:

   (a) 3π
   (b) π
   (c) 0
   (d) None of these

   Answer: (b)
   Solution: $\cos 3\pi = \cos \pi = -1$. So, $\cos^{-1}(-1) = \pi$, since the principal value of $\cos^{-1}$ lies in $[0, \pi]$.

3. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, and $xy = 1$, then:

   (a) $x = y$
   (b) $x = -y$
   (c) $x = 1$
   (d) $x + y = 1$

   Answer: (b)
   Solution: $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \frac{\pi}{4}$ implies $\frac{x+y}{1-xy} = 1$. Given $xy = 1$, we get denominator 0, so $\frac{x+y}{0}$ is undefined unless $x + y = 0 \Rightarrow x = -y$.

4. The value of $\cot^{-1}(\sqrt{3})$ is:

   (a) $\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$

(d) $\frac{\pi}{4}$

Answer: (a)
Solution: $\cot\left(\frac{\pi}{6}\right) = \sqrt{3} \Rightarrow \cot^{-1}(\sqrt{3}) = \frac{\pi}{6}$.

5. The expression $\cos^{-1}(-x) + \cos^{-1} x$ equals:

   (a) 0

   (b) π

   (c) $\frac{\pi}{2}$

   (d) 2π

   Answer: (b)
   Solution: From identity, $\cos^{-1}(-x) = \pi - \cos^{-1} x$, so $\cos^{-1}(-x) + \cos^{-1} x = \pi$.