**Case Study 4:**
In a summer enrichment program, a team of students led by Ravi were tasked to analyze the symmetry and periodicity properties of inverse trigonometric functions. While reviewing transformations and identities, the group discovered that understanding symmetry could greatly simplify the evaluation of inverse expressions. For instance, Ravi demonstrated that $\sin^{-1}(-x) = -\sin^{-1} x$, showing it was an odd function, and similarly found $\cos^{-1}(-x) = \pi - \cos^{-1} x$, an even property combined with a shift. They also worked on interpreting the graphs of these functions, particularly how domain restrictions determine the shape and output. Their teacher emphasized that recognizing these patterns leads to faster problem solving during competitive exams. Below are questions based on their analysis.

**MCQ Questions:**

1. The function $\cos^{-1} x$ is:

   (a) Even

   (b) Odd

   (c) Neither even nor odd

   (d) Periodic

   **Answer:** (a)
   **Solution:** $\cos^{-1}(-x) = \pi - \cos^{-1} x$, which is not equal to $\cos^{-1} x$ or $-\cos^{-1} x$, but $\cos^{-1} x$ is symmetric around $x = 0$, so it behaves like an even function in form.

2. The graph of $\sin^{-1} x$ is symmetric about:

   (a) x-axis

   (b) y-axis

   (c) Origin

   (d) Line $x = y$

   **Answer:** (c)
   **Solution:** Since $\sin^{-1}(-x) = -\sin^{-1} x$, the graph of $\sin^{-1} x$ is symmetric about the origin, which means it is an odd function.

3. The identity $\sin^{-1} x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ is valid for:

   (a) $x \in [-1, 1]$

   (b) $x \in (-\infty, \infty)$

   (c) $x \in (0, 1]$

   (d) $x \in [0, \infty)$

   **Answer:** (a)
   **Solution:** The expression under the square root $\sqrt{1 - x^2}$ must be real and defined. This implies $x^2 \leq 1$, or $x \in [-1, 1]$.

4. Which of the following is true for all $x \in [-1, 1]$?

   (a) $\sin^{-1} x = \cos^{-1} x$

(b) $\cos^{-1}(-x) = \cos^{-1} x$

(c) $\cos^{-1} x = \pi - \sin^{-1} x$

(d) $\cos^{-1} x + \cos^{-1} x = \pi$

**Answer:** (c)
**Solution:** $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \Rightarrow \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x = \pi - (\frac{\pi}{2} + \sin^{-1} x) = \pi - \sin^{-1} x$.

5. The value of $\sin^{-1}(-\frac{1}{2})$ is:

(a) $-\frac{\pi}{3}$

(b) $-\frac{\pi}{6}$

(c) $\frac{\pi}{6}$

(d) $\frac{\pi}{3}$

**Answer:** (b)
**Solution:** $\sin^{-1}(-x) = -\sin^{-1} x$, so $\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right) = -\frac{\pi}{6}$.