Case Study Based Questions
Chapter: Inverse Trigonometric Functions

Case Study Based Questions
Chapter: Inverse Trigonometric Functions

Case Study 3:

During a national-level math webinar, students from various schools are given a quiz to test their understanding of inverse trigonometric functions. One segment of the quiz focuses on simplifying composite inverse functions and interpreting their values graphically. Students are reminded that not all inverse trigonometric functions are defined over the entire real line and that choosing the correct principal value is essential. For example, knowing that $\tan^{-1}(\tan x) = x$ holds only when $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, helps avoid incorrect simplifications. Students also explore how to reverse trigonometric compositions like $\sin(\sin^{-1} x)$ and analyze when they hold true based on the domain of the inner function. The questions below reflect the types of logical and algebraic reasoning tested in this event.

MCQ Questions:

1. The value of $\sin(\sin^{-1} x)$ is:

   (a) x for all real x

   (b) x for $x \in [-1, 1]$

   (c) x for $x \in [0, 1]$

   (d) x for $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

   Answer: (b)
   Solution: Since $\sin^{-1} x$ is defined only for $x \in [-1, 1]$ and returns an angle $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, applying sin on it gives $\sin(\sin^{-1} x) = x$ only when $x \in [-1, 1]$.

2. The value of $\tan^{-1}(\tan x)$ is equal to x when x lies in:

   (a) $(0, \pi)$

   (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

   (c) $\left[0, \frac{\pi}{2}\right)$

   (d) $(-\pi, \pi)$

   Answer: (b)
   Solution: The identity $\tan^{-1}(\tan x) = x$ is valid only when x lies within the principal value branch of $\tan^{-1}$, i.e., $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

3. $\cos(\cos^{-1} x) = x$ holds true for:

   (a) $x \in [-1, 1]$
   (b) $x \in [0, \pi]$
   (c) $x \in [0, 1]$
   (d) $x \in [-1, 0]$

   Answer: (a)
   Solution: The inverse cosine function $\cos^{-1} x$ is defined for $x \in [-1, 1]$ and returns an angle in $[0, \pi]$. Applying cos gives back the original value x if x is within the domain of $\cos^{-1}$, i.e., $[-1, 1]$.

4. Which of the following is the correct identity?

   (a) $\tan^{-1} x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$
   (b) $\tan^{-1} x = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$
   (c) $\tan^{-1} x = \cot^{-1}\left(\frac{1}{x}\right)$
   (d) All of the above

   Answer: (d)
   Solution: All the given identities are valid representations of $\tan^{-1} x$ using inverse trigonometric identities and hold true within their respective domains.

5. The expression $\sin^{-1} x + \sin^{-1}(\sqrt{1-x^2})$ is equal to:

   (a) $\frac{\pi}{2}$ for $x \in [0, 1]$
   (b) π for $x \in [-1, 1]$
   (c) $\frac{\pi}{4}$ for $x = \frac{1}{\sqrt{2}}$
   (d) None of these

   Answer: (a)
   Solution: Let $x = \sin\theta$, so that $\sqrt{1-x^2} = \cos\theta$, then: $\sin^{-1} x + \sin^{-1}(\sqrt{1-x^2}) = \theta + \sin^{-1}(\cos\theta)$. But since $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$, this implies the full expression simplifies to $\frac{\pi}{2}$ for $x \in [0, 1]$.